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Mathematical Literacy of Hong Kong's 15-Year-Old Students in PISA *

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This article reports the performance of 15-year-old Hong Kong students in mathematical literacy in an international study called PISA 2000, in which Hong Kong has topped the list of 41 participating countries/regions, with a mean score significantly higher than all other countries except Japan and Korea. With the conceptual framework developed by PISA for the assessment of mathematical literacy, this survey on students' mathematics achievement is rather different from other international studies in that it focuses on students' competencies in applying knowledge and skills to understanding and solving problems and to making informed judgments and decisions in real-life situations. As PISA 2000 did not collect data related directly to classroom teaching and therefore could not provide an explanatory framework, it is speculated that the exceptionally good performance of Hong Kong students in PISA mathematical tasks may be attributed to their strengths in the basics of algebraic and geometric manipulations, as evidenced in previous international comparative studies. This article begins with a brief introduction to the rationale and general design of PISA and a detailed delineation of the conceptual framework of mathematical literacy. The overall performance of Hong Kong students, including

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gender differences, relative to the international mean performance will be outlined and analyzed in terms of different attributes of the PISA framework for mathematics. The final section discusses these PISA findings and derives some general implications for mathematics education in Hong Kong.

This article is a preliminary report of the mathematics performance of Hong Kong students in an international project known as the Programme for International Student Assessment (PISA), which was initiated and organized by the Organisation for Economic Co-operation and Development (OECD) in 2000. This international study aims at the comparison and evaluation of the effectiveness of education systems in the participating countries/regions. Educational indicators are to be derived from the study to assist governments and policy makers to evaluate and monitor the effectiveness of their education systems at the national level. More specifically, the study focuses on the assessment of how well the 15-year-old students who are approaching the end of compulsory education in the participating countries/regions have acquired the knowledge and skills essential for participating in society and meeting the challenges in a changing world.

Mathematics was one of the three domains of competencies assessed by PISA in 2000. Given the specific conceptual framework developed by PISA for the assessment of mathematical literacy, the PISA survey on students' mathematics achievement is rather different in essence from other international studies which focus mainly on assessing students' mathematics knowledge as defined by each of the national mathematics curricula of the participating regions and countries (Clarke, 2003). This article will begin with a brief introduction to the rationale and general design of PISA and then a detailed delineation of the conceptual framework of mathematical literacy, followed by sample items which illustrate the general design of the PISA assessment scheme in more concrete terms. The overall performance of Hong Kong students relative to the international mean performance will then be outlined and analyzed in terms of different attributes of the PISA framework for mathematics. Gender differences among Hong Kong students as observed in this study will also be presented. The final section discusses these PISA findings and derives some general implications for mathematics education in Hong Kong.

An Overview of PISA 2000

PISA aims to compare and evaluate the effectiveness of education systems across countries/regions of widely different school curricula, and addresses, more specifically, the following issues:

- How well are young adults prepared to meet the challenges of the future?
- Can they analyze, reason, and communicate their ideas effectively?
- Can they continue learning throughout their lives? (HKPISA Research Team, 2003)

Understandably, attempts to measure curriculum-based learning outcomes inevitably lead to the comparison of performance on the very limited, overlapping part of all the national/regional school curricula involved. So, instead of evaluating learning outcomes in terms of curricular content knowledge and skills, PISA focuses on students' competencies in applying knowledge and skills to understanding and solving problems and to making informed judgments and decisions in real-life situations. PISA attempts to test such "literacy" in broad concepts and skills and their applications, as OECD believes that such competencies to apply knowledge and skills are what can really help students adapt to future life in a changing world and support life-long learning. In its initial launch, PISA has studied three domains of literacy which are closely related to school learning, namely reading, mathematics, and science.

The first cycle of PISA, termed PISA 2000, was originally conducted in 2000 among 32 OECD countries/regions, which include the best economically developed countries in the world, and was later extended in 2002 to enroll a further group of 11 non-OECD countries/regions, known as PISA+. Hong Kong joined PISA+ and administered the PISA tests in February 2002. Whereas PISA generally measures literacy in three domains, PISA 2000 and PISA+ focused primarily on reading literacy. The PISA survey is to be repeated every three years, with the primary focus shifting to mathematics in 2003 and science in 2006, and back to reading in 2009. An additional domain — problem solving — was introduced in PISA 2003. In each cycle, each of the participating countries/regions invites about 5,000 students from at least 150 schools to take part in the survey according to a strictly monitored sampling scheme (for details, please refer to HKPISA Research Team, 2003). More than 200,000 15-year-old students from over 6,000 schools in 43 countries/regions took part in PISA 2000 and PISA+.[1]

Mathematical Literacy in the PISA Framework

In contrast to previous attempts of international comparison, PISA focuses on mathematical literacy which is concerned with students' capacity to draw upon their mathematical competencies to meet challenges of the future. It is concerned with students' capacities to analyze, reason, and communicate ideas effectively by posing, formulating, and solving mathematical problems in a variety of domains and situations. Mathematical literacy is defined in PISA as:

> the capacity to identify, to understand, and to engage in mathematics and make well-founded judgements about the role that mathematics plays, as needed for an individual's current and future private life, occupational life, social life with peers and relatives, and life as a constructive, concerned, and reflective citizen. (OECD, 2000, p. 50)

Thus, mathematical literacy is defined in terms of an individual's understanding of the role of mathematics and his/her capacity to engage in this discipline in ways that meet his/her needs. This puts the emphasis on the capacity to pose and to solve mathematical problems rather than to perform specific mathematical operations. Mathematical literacy is assessed in relation to content, process, and situations in which mathematics is used. The definition emphasizes the capacities to apply mathematical knowledge and skills across different situations and contexts in daily life. Accordingly, the PISA Consortium developed a framework for assessing mathematical literacy, which has three dimensions: mathematical content, mathematical processes, and mathematical situations and contexts (OECD, 1999, 2000).

Mathematical Content

PISA focuses on the assessment of students' capacities to apply mathematical knowledge and skills in real-life situations. However, this cannot be done in the absence of curriculum-based knowledge. In PISA, mathematical knowledge is organized around two aspects, namely *mathematical big ideas* and *mathematical curricular strands*. Mathematical big ideas, also referred to as "overarching concepts," represent clusters of relevant, connected concepts that appear in *real* situations and contexts. The PISA mathematics framework is innovative in specifying content in terms of these mathematical big ideas. Some of these big ideas include *chance, change and growth, dependency and relationships*, and *space and shape* (OECD, 1999, p. 48). Apart from this first level of content classification, PISA also classifies assessment items according to mathematical curricular strands, that is, the different content areas of mathematics in which many school mathematics curricula are usually organized. The main curricular strands in PISA include *number, measurement, estimation, algebra, functions, geometry, probability, statistics,* and *discrete mathematics* (OECD, 1999, p. 50). These mathematical curricular strands constitute, however, only a minor part in PISA. In other words, the construction and selection of assessment items is based primarily on the mathematical big ideas.

Given the limited mathematics assessment allowed in PISA 2000, only the major areas of “change and growth” and “space and shape” were assessed, because these areas can accommodate items from a variety of curricular strands without giving excessive emphasis to number skills. Within change and growth, PISA examined students’ ability to represent change and growth in comprehensible forms, to understand fundamental types of change, to recognize specific types of change as they occur, to apply these techniques to the outside world, and to control a changing world according to needs. Within space and shape, PISA assessed recognition of shapes and spatial forms in different representations, orientations or perspectives, and dimensions, and evaluated understanding of relationships between shapes and images or visual representations. Students’ abilities in observing similarities and differences in their analysis of components of spatial forms were involved. Aspects of geometry, trigonometry, and measurement were accommodated.

Mathematical Processes

The solution of any mathematical task involves a variety of mathematical processes. The PISA framework encompasses a set of general mathematical processes relevant to all levels of mathematics. General mathematical skills and competencies are conceptualized as associated with each of the following mathematical processes (OECD, 2002, pp. 82–83):

- mathematical thinking and reasoning
- mathematical argumentation
- mathematical communication
- modelling
- problem posing and solving
- representation
- using symbolic, formal and technical language and operations
- use of aids and tools

In daily life contexts, *real* mathematics demands the use of several mathematical skills listed above simultaneously. Developing items for each type of the skills separately would result in unnecessary, and somewhat artificial, compartmentalization of the mathematical process. Therefore, PISA does not use tasks that assess these competencies individually. Instead, PISA organizes these processes into three classes, defining the type of cognitive skills needed. These are (OECD, 2002, pp. 83–84):

- Competency Class 1: reproduction, definitions and computations
- Competency Class 2: connections and integration for problem solving
- Competency Class 3: mathematisation, mathematical thinking, generalisation and insight

Assessment items in PISA are structured and developed around these “competency clusters.” In general, these processes are in ascending order of difficulty. The classes, representing levels of mathematical competency, form a conceptual continuum, from simple reproduction of facts and computational skills, to the competency of making connections between different strands in order to solve simple real-world problems, and to the third class, which involves the “mathematisation” (Freudenthal, 1973; Treffers, 1987) of real-world problems and reflection on the solutions in the context of the problems, using mathematical thinking, reasoning, and generalization. The highest class can be considered as going to “the heart of mathematics and mathematical literacy.” However, it does not follow that a lower one must be mastered thoroughly in order to progress to an upper one. It should also be noted that since PISA 2000 allowed students to use calculators, the processes involved in some items might have been different to some extent for students who did not use calculators.

Mathematical Situations and Contexts

Doing and using mathematics in a variety of situations has played an important part in defining mathematical literacy, as it has been recognized that the choice of mathematical methods and presentations of results are often dependent upon the settings in which problems are framed. PISA assesses students' capacities to apply mathematical knowledge and skills *across* a variety of different situations and contexts, partly to minimize cultural bias. The situations in which the PISA mathematical tasks are set are categorized as *community*, *educational*, *occupational*, *personal*, and *scientific* (OECD, 2002, p. 16). The tasks resemble the kinds of problems people encounter in real life. In scientific contexts, proof of abstract conjectures, and generalizations of numeric or spatial patterns are commonly involved. Students' familiarity with these situations varies, reflecting the distance of the situation from the student, though it is not always clear how this distance may affect mathematical performance. In any case, PISA aims to ensure that tasks are based on "authentic" contexts which are likely to be found in the actual experiences and practices of the participants in a real-world setting. The introduction of authentic contexts does not imply the exclusion of interesting mathematical contexts. Neither does it exclude artificial fictional contexts based on the stylized representation of problems — such as the statistics of a fictitious town.

Types of Items in Mathematical Literacy Assessment

In PISA's mathematical assessment, several item formats are used (Table 1). These include *multiple-choice*, *closed-constructed response*, and *open-constructed response*. The multiple-choice format is widely used and noted for its economical nature. However, this type of question seems to be useful mainly for tasks demanding lower-level cognitive skills (Travers & Westbury, 1989). Closed-constructed response items are similar to multiple-choice items except that students are asked to provide the correct answer rather than to choose from given options. In this case, guessing is less likely to be a concern. For more cognitively demanding tasks and assessment of higher-order processes, open questions are preferred. Open-constructed response items require a more extended response from the student, and the process of producing a response is likely to involve higher-order cognitive activities. These items usually require the student to show steps taken or to explain how the answer was reached. This allows students to demonstrate their abilities by providing solutions at varied levels of mathematical complexity. In PISA, different item formats are usually linked to a common stimulus material (OECD, 1999). Typically, a few multiple-choice or closed-constructed items are followed by open-constructed items. This arrangement allows realistic tasks to be devised and the complexity of real-life situations to be reflected. Moreover, this allows effective use of the assessment time by minimizing the time needed for switching stimuli.

Table 1. Distribution of Items by the PISA Framework Dimensions for the Assessment of Mathematical Literacy

	Total no. of items	No. of multiple-choice Items	No. of closed-constructed items	No. of open-constructed items
Distribution of items by mathematical big idea (content)				
Change and growth	17	5	9	3
Space and shape	14	5	9	—
Total	**31**	**10**	**18**	**3**
Distribution of items by curricular strand (content)				
Algebra	5	—	4	1
Functions	5	4	—	1
Geometry	8	3	5	—
Measurement	6	2	4	—
Number	1	—	1	—
Statistics	6	1	4	1
Total	**31**	**10**	**18**	**3**
Distribution of items by competency class (process)				
Class 1	10	4	6	—
Class 2	19	6	11	2
Class 3	2	—	1	1
Total	**31**	**10**	**18**	**3**
Distribution of items by situation (context)				
Community	4	—	2	2
Educational	6	2	3	1
Occupational	3	1	2	—
Personal	11	5	6	—
Scientific	7	2	5	—
Total	**31**	**10**	**18**	**3**

Proficiency in Mathematical Literacy

Mathematical performance in PISA 2000 is measured on a single scale, which has a mean of 500 points and a standard deviation of 100 points, with about two-thirds of students across the OECD countries/regions scoring between 400 and 600 points. The scale measures students' abilities to apply mathematical knowledge and skills listed in the previous section (OECD & UNESCO, 2003). It should be noted that the number of items used in PISA 2000 was only 31. The small number of items does not allow construction of standardized scores for the three competency classes without compromising the precision of measurement. However, this can be done in PISA 2003, when mathematics is the major assessment domain.

When scaling the mathematical literacy scores, item difficulties are taken into account by using techniques of item response theory. The items can be classified into three difficulty levels: highest, middle, and lowest. At the highest level, around 750 points, students should be able to interpret and formulate mathematical problems in complex situations that involve several steps. These students display higher-order cognitive processes such as generalization, reasoning, and argumentation to communicate results. At the middle level, around 570 points, students should be able to work with given strategies, models, propositions, or representations. They can solve mathematical problems that involve a few steps. At the lowest level, around 380 points, students should be able to apply simple computational skills and reproduce basic mathematical facts or processes.

Qualitatively, task difficulty is determined by the following criteria:

- *The number and complexity of processing or computational steps involved in the tasks*. Tasks range from single-step problems requiring students to recall and to reproduce basic mathematical facts or to complete simple computations to more complex problems calling for advanced mathematical knowledge and complex decision-making, information processing and problem-solving and modelling skills.
- *The requirement to connect and integrate material*. The simplest tasks typically require students to apply a single representation or technique to a single piece of information. More complicated tasks require students to integrate more than one piece of information using different representations, or different mathematical tools or knowledge in a sequence of steps.

- *The requirement to represent and interpret material and to reflect on situations and methods.* Such tasks range from recognising and using a familiar formula to the formulation, translation or creation of an appropriate model within an unfamiliar context, and the use of insight, reasoning, argumentation and generalisation. (OECD & UNESCO, 2003, p. 93)

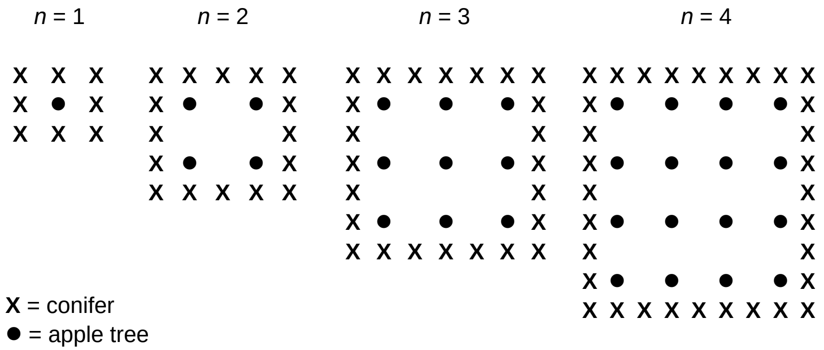
Sample Mathematical Items

The tasks used to assess mathematical literacy in PISA are wide-ranging in terms of the types of situation and levels of difficulty. These tasks are set in “units,” usually with two or more items relating to a given scenario described by a piece of text with or without accompanying diagrams. Only a small sample of the mathematical items was released for use in reporting PISA 2000 results (OECD, 2002). Some of these sample items are included in this article to demonstrate the various aspects of the PISA framework as delineated, the schematic procedure in categorizing students’ response, and the range of complexity involved in the assessment tasks in general. In each of the included items, the task difficulty is provided, together with the average performance of Hong Kong students as compared with the international average.

Sample Task 1: Apples

A farmer plants apple trees in a square pattern. In order to protect the trees against the wind he plants conifers all around the orchard.

Here you see a diagram of this situation where you can see the pattern of apple trees and conifers for any number (n) of rows of apple trees:



Question 1: Apples

Complete the table:

n	*Number of apple trees*	*Number of conifers*
1	**1**	**8**
2	**4**	*16*
3	*9*	*24*
4	*16*	*32*
5	*25*	*40*

Score 2:

Answers which show all 7 entries correct.

Task Difficulty: Middle, 548
Content: Change and growth
Process: Competency class 2
Context: Educational

Hong Kong mean % correct	OECD mean % correct
78	50

Students are given a hypothetical scenario involving planting an orchard of apple trees in a square pattern, with a row of protective conifer trees around the square. They are asked to complete a table of values generated by the functions that describe the number of trees as the size of the orchard is increased. This question requires students to interpret a written description of a problem situation, to link this to a tabular representation of some of the information, to recognize a pattern and then to extend this pattern. Students need to work with given models and to relate two different representations (pictorial and tabular) of two relationships (one quadratic and one linear) in order to extend the pattern.

Question 2: Apples

There are two formulae you can use to calculate the number of apple trees and the number of conifers for the pattern described above:

Number of apple trees = n^2
Number of conifers = $8n$
where n is the number of rows of apple trees.

There is a value of n for which the number of apple trees equals the number of conifers. Find the value of n and show your method of calculating this.

Score 1:

Answers which give $n = 8$, with the algebraic method explicitly shown.
Answers which give $n = 8$, but no clear algebra is presented, or no work shown.
Answers which give $n = 8$, using other methods, e.g., using pattern expansion or drawing.

Task Difficulty: Middle to Highest, 655
Content: Change and growth
Process: Competency class 2
Context: Educational

Hong Kong mean % correct	OECD mean % correct
56	25

This task requires students to interpret expressions containing words and symbols, and to link different representations (pictorial, verbal and algebraic) of two relationships (one quadratic and one linear). Students have to find a strategy for determining when the two functions will have the same solution (for example, by trial and error, or by algebraic means), and to communicate the result by explaining the reasoning and calculation steps involved.

Question 3: Apples

Suppose the farmer wants to make a much larger orchard with many rows of trees. As the farmer makes the orchard bigger, which will increase more quickly: the number of apple trees or the number of conifers? Explain how you found your answer.

Score 2:

Answers which are correct (apple trees) AND which give some algebraic explanations based on the formulae n^2 and $8n$.

Score 1:

Answers which are correct (apple trees) AND are based on specific examples or on extending the table.
Answers which are correct (apple trees) and show SOME evidence that the relationship between n^2 and $8n$ is understood, but not so clearly expressed as in Score 2.

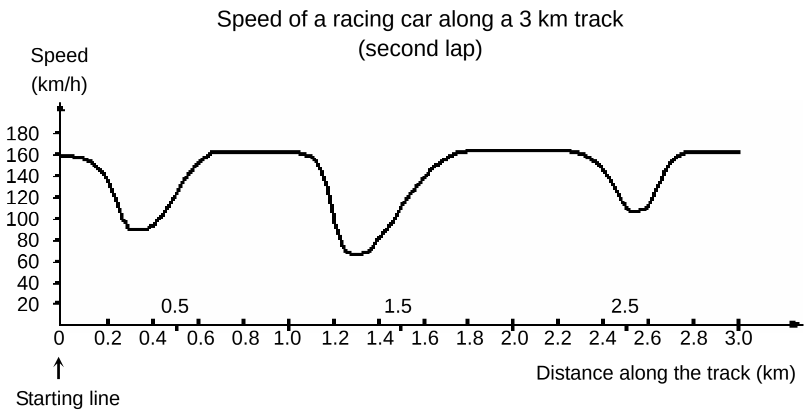
Task Difficulty: Highest, 723
Content: Change and growth
Process: Competency class 3
Context: Educational

Hong Kong mean % correct	OECD mean % correct
27	13

This task requires students to show insight into mathematical functions by comparing the growth of a linear function with that of a quadratic function. Students are required to construct a verbal description of a generalized pattern, and to create an argument using algebra. Students need to understand both the algebraic expressions used to describe the pattern and the underlying functional relationship, in such a way that they can see and explain the generalization of these relationships in an unfamiliar context. A chain of reasoning is required, and communication of this in a written explanation.

Sample Task 2: Speed of Racing Car

This graph shows how the speed of a racing car varies along a flat 3-kilometre track during its second lap.



Question 1: Speed of Racing Car

What is the approximate distance from the starting line to the beginning of the longest straight section of the track?

A. 0.5 km
B. 1.5 km
C. 2.3 km
D. 2.6 km

Score 1:
B. 1.5 km

Task difficulty: Middle, 492
Content: Change and growth
Process: Competency class 2
Context: Scientific

Hong Kong mean % correct	OECD mean % correct
N/A	67

* N/A: This item was deleted from subsequent analysis.

This task requires students to interpret a graphic representation of a physical relationship (distance and speed of a car traveling on a track of unknown shape). Students need to interpret the graph by linking a verbal description with two particular features of the graph (one simple and straightforward, and one requiring a deeper understanding of several elements of the graph and what it represents), and then to identify and read the required information from the graph, selecting the best option from given alternatives.

Question 2: Speed of Racing Car

Where was the lowest speed recorded during the second lap?
A. at the starting line
B. at about 0.8 km
C. at about 1.3 km
D. halfway around the track

Score 1:
C. at about 1.3 km

Task difficulty: Lowest, 403
Content: Change and growth
Process: Competency class 1
Context: Scientific

Hong Kong mean % correct	OECD mean % correct
90	83

The question requires students to read information from a graph representing a physical relationship (speed and distance of a car). Students need to identify one specified feature of the graph (the display of speed), to read directly from the graph a value that minimizes the feature, and then to select the best match from among given alternatives.

Question 3: Speed of Racing Car

What can you say about the speed of the car between the 2.6 km and 2.8 km marks?
A. The speed of the car remains constant.
B. The speed of the car is increasing.

C. The speed of the car is decreasing.
D. The speed of the car cannot be determined from the graph.

Score 1:
B. The speed of the car is increasing.

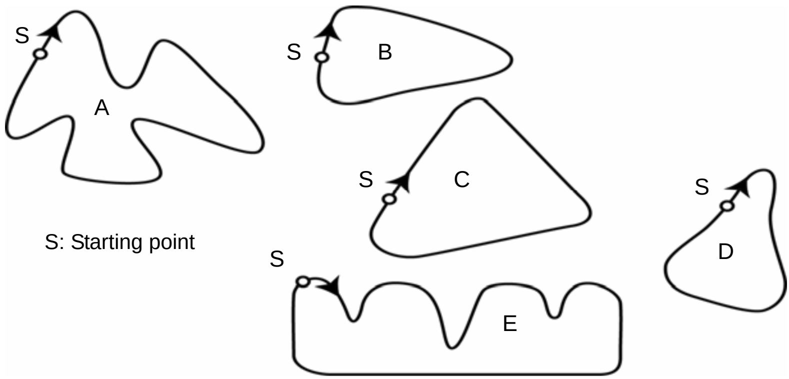
Task difficulty: Lowest, 413
Content: Change and growth
Process: Competency class 1
Context: Scientific

Hong Kong mean % correct	OECD mean % correct
82	83

This task requires student to read information from a graph representing a physical relationship (speed and distance of a car). Students need to identify the place in the graph referred to in a verbal description to recognize what is happening to the speed of the vehicle at that point, and then to select the best matching option from among given alternatives.

Question 4: Speed of Racing Car

Here are pictures of five tracks:



Along which one of these tracks was the car driven to produce the speed graph shown earlier?

Score 1:
B

Task difficulty: Highest, 655
Content: Change and growth
Process: Competency class 2
Context: Scientific

Hong Kong mean % correct	OECD mean % correct
36	29

This task requires students to understand and interpret a graphic representation of a physical relationship (speed and distance of a car) and to relate it to the physical world. Students need to link and integrate two very different visual representations of the progress of a car around a racetrack. Students have to identify and select the correct option from among given challenging alternatives.

Overall Performance of Hong Kong Students in Mathematical Literacy

Among the 41 countries/regions participating in PISA 2000 and PISA+, Hong Kong students' performance in mathematical literacy is at the top, with a mean score of 560 and a standard error of 3.3.[2] As shown in Table 2, Hong Kong scored significantly higher than all other countries except Japan (mean score 557) and Korea (mean score 547). Whereas Hong Kong has a score slightly higher than those of Japan and Korea, the differences between the scores are not statistically significant.

Another way to assess the performance of Hong Kong students in mathematical literacy is to compare the scores of Hong Kong students at different levels of ability with the corresponding student groups of the OECD countries. Table 3 and Figure 1 present the statistics in tabular and graphic forms respectively.

Among the OECD countries/regions, the best 5% of students achieved 655 points or more in the PISA mathematics assessment. The best 10% attained 625 points or above, and the best 25% reached at least 571 points. At the lower end of the mathematical literacy scale, 75% of the students achieved at least 435 points, 90% reached 367 points, and 95% attained 326 points or above. At each percentile, the average score of Hong Kong students is always higher than the OECD average, indicating that Hong Kong students at all levels of ability generally achieved a higher level of mathematical literacy than the international average. As can be seen in Figure 1, the difference in scores at different percentiles is quite uniform, indicating that lower and higher achievers in Hong Kong are doing equally well compared to their international counterparts.

Table 2. Comparison of Mean Performance on the Mathematical Literacy Scale

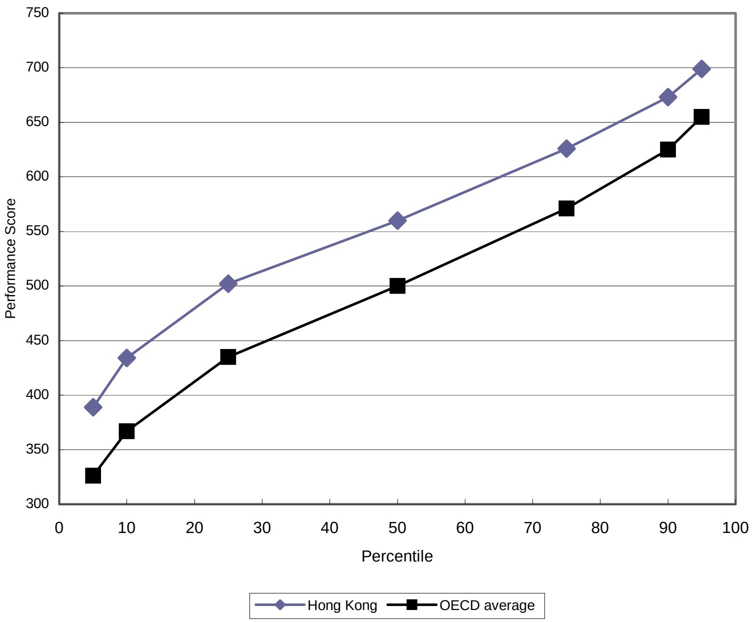
Country/region	Mean	*SE*	Remark
Hong Kong	**560**	**3.3**	—
Japan	557	5.5	#
Korea	547	2.8	#
New Zealand	537	3.1	✶
Finland	536	2.2	✶
Australia	533	3.5	✶
Canada	533	1.4	✶
Switzerland	529	4.4	✶
United Kingdom	529	2.5	✶
Belgium	520	3.9	✶
France	517	2.7	✶
Austria	515	2.5	✶
Denmark	514	2.4	✶
Iceland	514	2.3	✶
Liechtenstein	514	7.0	✶
Sweden	510	2.5	✶
Ireland	503	2.7	✶
Norway	499	2.8	✶
Czech Republic	498	2.8	✶
United States	493	7.6	✶
Germany	490	2.5	✶
Hungary	488	4.0	✶
Russian Federation	478	5.5	✶
Spain	476	3.1	✶
Poland	470	5.5	✶
Latvia	463	4.5	✶
Italy	457	2.9	✶
Portugal	454	4.1	✶
Greece	447	5.6	✶
Luxembourg	446	2.0	✶
Israel	433	9.3	✶
Thailand	432	3.6	✶
Bulgaria	430	5.7	✶
Argentina	388	9.4	✶
Mexico	387	3.4	✶
Chile	384	3.7	✶
Albania	381	3.1	✶
Macedonia	381	2.7	✶
Indonesia	367	4.5	✶
Brazil	334	3.7	✶
Peru	292	4.4	✶

Note: # denotes score that is not significantly higher or lower than that of Hong Kong;
✶ denotes score that is significantly lower than that of Hong Kong.

Table 3. Student Scores of Hong Kong and OECD Countries/regions in Mathematical Literacy at Different Percentiles

	Hong Kong	OECD average
5th percentile	389	326
10th percentile	434	367
25th percentile	502	435
50th percentile	560	500
75th percentile	626	571
90th percentile	673	625
95th percentile	699	655

Figure 1. Student Scores in Mathematical Literacy at Different Percentiles



Performance of Hong Kong Students Compared With Other Countries/regions

As can be seen above, Hong Kong students outperformed their counterparts in OECD countries, and this difference in scores, on average, is substantial at various ability levels. A particularly meaningful way to understand the qualitative aspect of this difference in performance is to compare it with other countries on the various dimensions constituting the PISA framework for assessing mathematical literacy. Table 4 shows the average percentage scores of students of OECD countries and of Hong Kong on the mathematical items by content, process, and context respectively.[3] Note that the content dimension can be delineated in terms of either "mathematical big ideas" or the usual "curricular strands." It can be observed that for all categories, the Hong Kong average is greater than the OECD average.

Table 4. Comparison of Student Scores of Hong Kong and OECD Countries/regions on Various Dimensions of the PISA Framework

	Hong Kong average (%)	OECD average* (%)
Distribution of items by mathematical big idea (content)		
Change and growth (16)	57	44
Space and shape (13)	62	50
Distribution of items by curricular strand (content)		
Algebra (5)	55	28
Functions (4)	62	55
Geometry (8)	65	54
Measurement (6)	56	41
Number (0)	N/A	N/A
Statistics (6)	58	50
Distribution of items by competency class (process)		
Class 1 (9)	75	65
Class 2 (18)	55	41
Class 3 (2)	27	14
Distribution of items by situation (context)		
Community (3)	51	47
Educational (6)	56	43
Occupational (3)	68	46
Personal (10)	58	45
Scientific (7)	64	51

Notes: The numbers in parentheses indicate the numbers of items for the designated categories.

* Two items were deleted from the Hong Kong data set for subsequent analysis. This OECD average is calculated based on the 29 corresponding items.

N/A: The item was deleted from the Hong Kong data set by OECD.

A closer look at the figures reveals that the difference between the two columns is generally quite substantial for most categories, and the difference is particularly big for items on algebra and on competency class 3. In terms of curricular strands, Hong Kong students outperformed the OECD average on all topics appreciably, and their performance is exceptionally high in algebra. This may indicate the particular strength of Hong Kong students in algebraic manipulations, probably because of the strong emphasis on algebra in the Hong Kong mathematics curriculum and the usual, intensive drill-and-practice mode of learning associated with algebraic topics. On the other hand, the average item score decreased, as can be expected, with the level of competency class for both OECD and Hong Kong students. That is to say, both groups of students performed much better on items involving reproduction, definitions, and computations than those involving higher-order mathematical skills. In this regard, Hong Kong students also showed an impressive performance on items of competency class 3, with an average percentage score of 27 as compared to the OECD average of 14.

Furthermore, when the item scores were analyzed in terms of the item types (i.e., multiple-choice, closed-constructed response, and open-constructed response type), the average item score, as expected, decreased with the item type in the order of multiple-choice, closed-constructed response, and open-constructed response for both groups of students. Again, Hong Kong students excelled on all categories. The details are shown in Table 5.

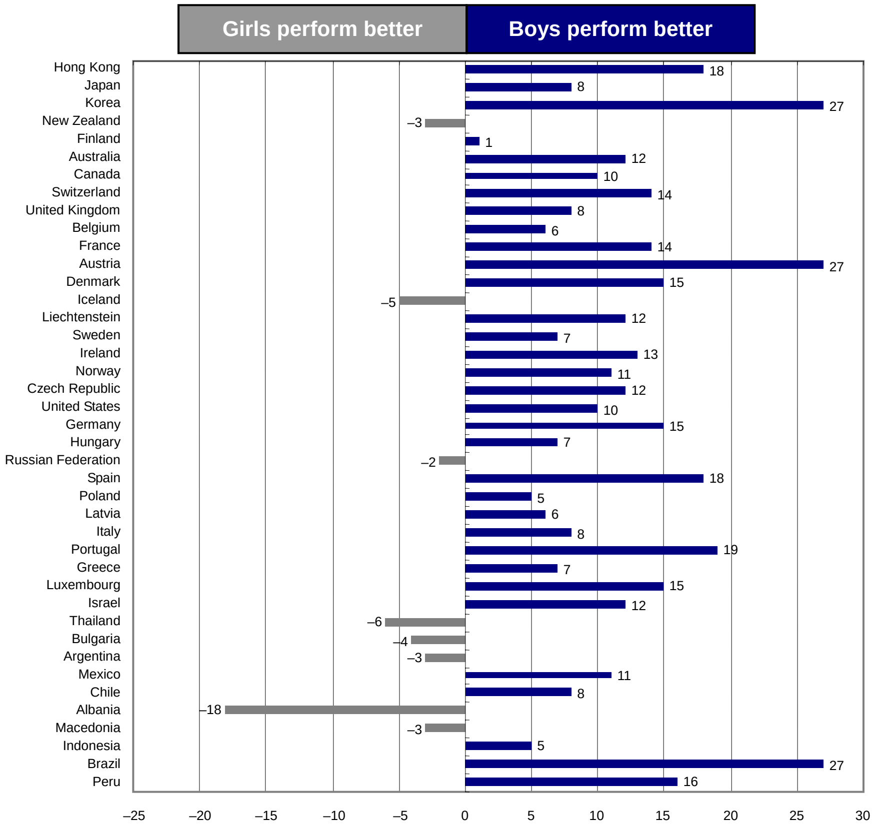
Table 5. Comparison of Students' Mathematics Scores of Hong Kong and OECD Countries/regions by Item Types

Item type	Hong Kong (%)	OECD average (%)
Multiple-choice	66	57
Closed-constructed response	60	44
Open-constructed response	37	29

Gender Differences in Mathematical Literacy of Hong Kong Students

Figure 2 shows the gender differences in mathematical performance among the participating countries/regions in the study. The general

Figure 2. Gender Differences in Mathematical Performance



Note: The countries/regions are sequenced according to the overall mathematical performance.

pattern is that there are statistically significant differences in about half of the participating countries/regions, in all of which males performed better. The biggest gender differences are found in Korea, Austria and

Brazil. On the other hand, Albania presents a unique, opposite case, in which girls achieved significantly higher scores than boys. In comparison, Hong Kong is among the group of participating countries/regions of moderate gender differences in mathematical performance. However, there is no consistent pattern of gender differences in mathematical literacy among the participating countries/regions in the PISA study. Such gender differences, if they exist, may suggest that there are underlying features in the education systems or societies and cultures which have favored one gender more than the other in school (Fennema & Leder, 1990; Leder, 1992, 1996).

In Hong Kong, boys performed significantly better than girls by 18 points on average. This is in line with the general impression and belief among local mathematics teachers in secondary schools. However, a detailed analysis shows that the boys' dominant position decreases along the level of performance in descending order, as shown in Table 6. The difference in mathematical performance between boys and girls narrows for students at lower percentiles, and in fact for students below the 10th percentile, girls have performed better than boys. This overall pattern of gender difference among Hong Kong students is represented in Figure 3.

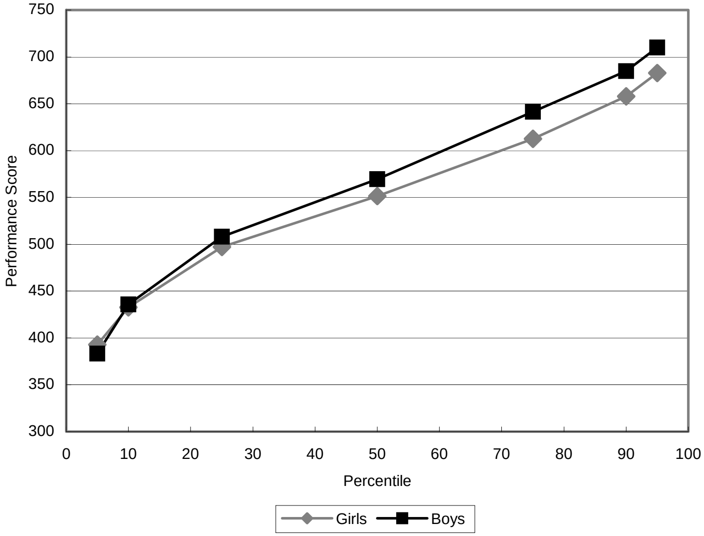
Table 6. Performance of Hong Kong Girls and Boys on Mathematical Literacy at Different Percentiles

	Girls	Boys
5th percentile	393	383
10th percentile	433	436
25th percentile	497	508
50th percentile	551	569
75th percentile	613	642
90th percentile	658	685
95th percentile	683	710

This trend of gender difference in mathematical performance is consistent with the dominance of boys in the mathematics stream at the matriculation level, as more boys are in the high performance group and can therefore compete more successfully than girls for the limited number of places in matriculation classes.

The gender difference found in the present PISA study is even more noteworthy when the percentage scores are analyzed in terms of the various dimensions constituting the PISA framework for assessing

Figure 3. Comparison of Scores of Hong Kong Girls and Boys in Mathematical Literacy



mathematical literacy. Tables 7 and 8 show the details of the statistics concerned. Boys performed significantly better than girls in both content domains of "change and growth" and "space and shape." In terms of curricular strand, again, boys significantly outperformed girls in all topics except statistics, which requires perhaps the least mathematical sophistication in relative terms. When scores are analyzed in terms of the competency classes, the average item score decreases, as expected, with the level of competency class for both boys and girls. That is to say, both boys and girls performed much better on items involving reproduction, definitions, and computations than those involving higher-order mathematical skills. At each competency class level, boys achieved scores significantly higher than those of girls. As for the comparison of mathematical scores between girls and boys by the type of contexts, boys achieved an average score greater than or equal to that of girls in every context type, and except for the community context, the difference was statistically significant for all other context types.

Table 7. Gender Differences in Mathematical Literacy of Hong Kong Students on the Various Dimensions of the PISA Framework

Dimension	Girls	Boys	Difference between boys and girls	Overall
Distribution of items by mathematical big idea (content)				
Change and growth	54	60	**	57
Space and shape	59	65	**	62
Distribution of items by curricular strand (content)				
Algebra	52	58	**	55
Functions	57	66	**	62
Geometry	62	68	**	65
Measurement	53	59	**	56
Number	N/A	N/A	—	N/A
Statistics	57	58	—	58
Distribution of items by competency class (process)				
Class 1	73	77	**	75
Class 2	52	58	**	55
Class 3	25	29	**	27

** Difference of the means is significant at the 0.001 level.

Table 8. Gender Differences in Mathematical Literacy of Hong Kong Students on the Dimension of Context of the PISA Framework

Distribution of items by situation (context)	Girls	Boys	Difference between boys and girls	Overall
Community	51	51	—	51
Educational	53	59	**	56
Occupational	65	71	**	68
Personal	55	61	**	58
Scientific	61	66	**	64

** Difference of the means is significant at the 0.001 level.

Lastly, when the item scores are analyzed in terms of item type (i.e., multiple-choice, closed-constructed response, and open-constructed response type), the average item score decreases with the item type in the order of multiple-choice, closed-constructed response, and open-constructed response for both boys and girls. This trend is quite understandable in view of the nature of the responses required from the students. Furthermore, boys outperformed girls on all three categories, though the difference is only statistically significant for the two categories of multiple-choice and closed-constructed response type. The details are shown in Table 9.

Table 9. Comparison of Mathematics Scores of Hong Kong Girls and Boys by Item Types

Item type	Girls	Boys	Difference between boys and girls
Multiple-choice	62	69	**
Closed-constructed response	57	62	**
Open-constructed response	35	38	—

** Difference of the means is significant at the 0.001 level.

The above analysis indicates unequivocally the substantial gender differences in various aspects of mathematical performance in Hong Kong, ranging from mathematical content to levels of mathematical processes. This difference is at a moderate level in the international comparison. The overall dominance of boys in mathematical learning must have originated from contextual or cultural factors in the learning environment and in the community life which affect gender differences in school performance in general, and mathematical performance in particular (Jungwirth, 2003). This issue of gender differences in performance, owing perhaps to subtle inequalities between the genders in the local mathematics classroom, warrants the attention of mathematics teachers and educators. Unfortunately, research on gender differences in local mathematical learning has been relatively scarce (Leung, Lam, Mok, Wong, & Wong, 1999; N. Y. Wong, Lam, Leung, Mok, & Wong, 1999). It is therefore recommended that more local research efforts should be directed to the investigation of gender differences, and this should be included as an important item in the agenda of the next mathematics curriculum review.

Discussion

As seen in the previous sections, Hong Kong students have outperformed the other countries/regions in terms of the mathematical literacy framework of PISA 2000, with a mean score that is the highest and significantly higher than all other countries except Japan and Korea. This high achievement is also evidenced by the fact that their scores are uniformly higher than the OECD means across all percentile groups. This result, while still being better than expected, may not be regarded as a complete surprise in view of the earlier findings in the Third International Mathematics and Science Study (TIMSS) of 1995 (Leung & Wong, 1997a, 1997b). In the TIMSS comparison, Hong Kong was already among the countries/regions of top performance, being the fourth in the “league table” and ranked lower than Singapore, Japan, and Korea. In that same study, it was also found that Hong Kong students performed less well in open problems and performance assessment tasks (Leung, 2003; M. P. H. Wong, 1995).

Given this background, the exceptionally good performance of Hong Kong students in the so-called authentic mathematical tasks of the present PISA assessment scheme is therefore, to a certain extent, contrary to the “received wisdom” among many local mathematics teachers so far. Our teachers tended to believe that most of our students are good at mathematical problems of the textbook types, but not good, if not at all weak, at mathematical applications, especially those problems that require some understanding of real-life situations (Lam, Wong, & Wong, 1999; N. Y. Wong et al., 1999; N. Y. Wong, Marton, Wong, & Lam, 2002). Local teachers generally complain, as do many of their counterparts in other countries/regions, that word problems are fatal tasks in the mathematics classroom for most of the students, not to mention those problems which need realistic considerations on top of the mathematics involved. For instance, it is quite common to hear teachers say that their students just put down a negative value as an answer for the age of a person, or a non-integral value as an answer for the number of vehicles needed in a transportation task, when such values are correct, legitimate mathematical solutions to the equations obtained. Given the verbal nature of the PISA mathematics assessment items as well as the necessity to understand real-life situations involved in most of these items, the aforementioned “received wisdom” is under challenge, at least from an international comparative perspective (K. M. P. Wong & Law, 2003).

While there is no place for complacency even though Hong Kong has topped the list in PISA 2000, it is perhaps a good time for our mathematics teachers to feel relieved and to reflect upon whether we have been too harsh to our students and too demanding in our teaching, when many other countries/regions, with their more lively or more liberal approaches adopted for teaching school mathematics, cannot outperform us. After all, ***if*** these PISA results are considered valid, it may indicate that mathematical literacy as defined by PISA is closely, perhaps more closely than most have anticipated, hinged upon basic knowledge and skills delivered in conventional mathematics teaching; good performance of Hong Kong students in PISA mathematical tasks has to be traced back to their strengths in the basics of algebraic and geometric manipulations. This should not be surprising, and when TIMSS results are also taken into consideration, should be, in fact, a very sensible conclusion, if not for the *Zeitgeist* in education of the twenty-first century of unreserved emphasis or bias toward the enhancement of higher-order thinking and process abilities as the single valuable aim, the Holy Grail, of mathematics education. Maybe local mathematics teachers and educators should be thereby reminded that our existing strengths in cultivating basic mathematical knowledge and skills should never be sacrificed for the much more illusive, sole pursuit of the higher-order abilities proposed in the ungrounded, high-sounding agenda of the curriculum reform advocated by the local authorities. Of course, as the italic "if" in front has indicated, this conclusion should be taken with a pinch of salt, because we are discussing an international comparison based on a mere collection of 29 mathematical items distributed rather unevenly in different curricular strands involving only two so-called mathematical big ideas (see Table 1). Furthermore, each of the participating students just took a small, selected portion of these items in the PISA 2000 survey.

If one wants to repudiate this conclusion, another line of attack is to reflect upon the validity of the test items designed for assessing the mathematical literacy defined. Admittedly, this goes greatly beyond the scope of the present article, and the capacity of the author too. Except for a selected few, PISA items are confined to the perusal of research team members only, and we have to believe in the experts taking part in the design of these test items. Unlike test items of clearly defined curricular contents in TIMSS, PISA mathematics items based on real-life situations are nonetheless intertwined with essential cultural elements. For obvious reasons, there is yet not enough room for an open discussion on whether the design of these items really meets the strict demand of the definition in an international context of cross-cultural comparison. How this job can be accomplished appropriately and satisfactorily is, in the author's humble opinion, quite intractable, and unfortunately, can neither be subject to open scrutiny at the moment. Were most of the PISA 2000 test items actually quite similar to what Hong Kong students already had at school? To this question, the research community at large has to wait. Furthermore, since PISA 2000 did not collect data pertaining to the input side of mathematics teaching at school save demographic data, it is not wise at this stage to draw conclusive statements about the underlying causes of observed differences in students' performance, or else, any such statement would be highly speculative. For more substantial answers and grounded insights into questions raised in this article, we have to, though unwillingly, wait for the analysis of the data collected in the second cycle of PISA (i.e., PISA 2003), in which mathematical literacy is the primary focus of the survey and a more elaborated scale of mathematical competencies can be constructed.

Notes

1. There were altogether 43 countries/regions taking part in the first cycle of PISA, i.e., PISA 2000 and PISA+. However, because the response rate of the Netherlands was below the standard specified by OECD, and data from Romania were unavailable when the present analysis was conducted, therefore only 41 countries/regions will be mentioned in this report hereafter.
2. Participating in this PISA+ survey were altogether 4,405 Hong Kong students from 140 local schools, consisting of 2,197 boys and 2,208 girls (HKPISA Research Team, 2003, p. 15).
3. The PISA 2000 mathematics items were divided into four clusters appearing in different combination in nine different PISA assessment booklets to be taken by sampled students. Each student answered one particular booklet during the two-hour assessment session. Therefore, each student just answered a portion of the entire collection of test items, some more, some less, and none of them answered all of the items (for details, please refer to HKPISA Research Team, 2003). So, whenever analysis was conducted in terms of test items in this report, all statistics shown in the tables were computed by an item level approach. That is, statistics for a particular test item were obtained in the computation by pooling scores of all those students who had taken the item in question.

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