# Mathematics Background and Preparation

MSc Mathematics Education
The Chinese University of Hong Kong

## Introduction

This is a reference document for potential students of the Master of Science Programme in Mathematics Education of The Chinese University of Hong Kong. It may be used for the following purposes.

- In the consideration whether you should apply for admission into this programme, you may like to know whether you have sufficient mathematics background.
- After you have been admitted, you may like to prepare or refresh the mathematics needed for the study of this programme.

## Topics

Some topics are listed below. These topics form the minimum required foundation. A student with only such background will need to spend consideration effort in catching up the course materials.

The topics are divided into several categories. In each category, candidates are expected to be very familiar with half of the topics, and know the other half with not much reviewing study.

### General

- Set notations, polynomials and rational functions, trigonometric functions and identities, logarithm and exponential, coordinate geometry and standard conics, permutation and combination, probability, simple inequalities.
- Rigorous logics and set language, equivalence and quotient, parametrized curves and loci, hyperbolic functions, rotated conics, inequalities involving roots and absolute value, identities involving permutation and combination, complex numbers, algebra and geometry of complex numbers.

  References. Any mathematics text book used for Advanced Level Pure Mathematics.

### Calculus

- Limits, differentiation, maximum and minimum, curve sketching, integration, application of integration.

- Sequence, continuity, infinite series, power series, Taylor's expansion, multivariable functions, partial derivatives, gradient, maximum and minimum, multiple integrals.

- Rigorous $\varepsilon$-$\delta$ argument, differentiability function of one or several variables, Mean Value Theorem, differential, Inverse or Implicit Function Theorem, Green's and Stokes' Theorems.

  REFERENCES.

  1. Salas, Etgen, and Hille, *Calculus: one and several variables*, Wiley.
  2. Thomas, Weir, Hass, and Giordano, *Thomas' Calculus: Early Transcendentals*, Addison-Wesley.
  3. Edwards and Penny, *Calculus: Early Transcendentals*, Prentice Hall.

### Linear Algebra

- Solving linear equations, Gaussian elimination, matrix operations, determinant.

- Vector spaces, matrix transformation, Gram-Schmidt process, eigenvalues and eigenvectors, numerical applications.

- Linear transformations, kernel and rank, isomorphisms, inner products, diagonalizability, normal forms; groups, rings, fields.

  REFERENCES.

  1. Howard Anton, ..., *Elementary Linear Algebra (with applications)*, Wiley.
  2. Steve Leon, *Linear Algebra with Applications*, Prentice Hall.

## Questions

The listed questions are not a type of assessment test. If you intend to apply for this programme, you should be able to formulate the method or strategy for a question within 2 or 3 minutes, though it may take more time to actually work it out.

1. Give an example of functions $f: X \to Y$ and $g: Y \to Z$ such that $g \circ f$ is onto be neither $f$ nor $g$ is so.

2. Suppose a country has only \$2-note and \$5-note. Show that this system is enough for any dollar value of 4 or more.

3. Find the greatest common divisor of the polynomials $p(x) = x^4 - 2x^3 - 4x^2 + 4 - 3$ and $q(x) = 2x^3 - 5x^2 - 4x + 3$.

4. Explore the situation of solution(s) for the system

$$\begin{aligned} ax + y + z &= 1 \\ x + ay + z &= a \\ x + y + az &= a^2. \end{aligned}$$

5. Express the inverse $(I - A)^{-1}$ in terms of the matrix $A$ if there is an integer $n$, $A^n = 0$.

6. Find the eigenvalues and corresponding eigenvectors for the matrix

$$A = \begin{pmatrix} 3 & 2 & -1 \\ -2 & -2 & 2 \\ 3 & 6 & -1 \end{pmatrix}$$

7. Let $f(x)$ be a real polynomial such that it is divisible by $f'(x)$. Find its general form.

8. Determine the type of conics given by $73x^2 - 72xy + 5xy^2 - 30x - 40y - 75 = 0$.

9. Find and classify the critical points of the function $f(x, y) = e^2\left(x^2 - y^2\right)$.

10. Let $E$ be an ellipsoid defined by $rx^2 + y^2 + 4z^2 = 16$. The temperature on the surface of this ellipsoid is given by

$$T(x, y, z) = 8x^2 + 4yz - 16z + 600\,.$$

Find the hottest point on $E$.

11. Let $f(x, y) = 16 - x^2 - y^2$ be defined on the region $S \subset \mathbb{R}^2$ bounded by the curves $y^2 = x$ and $y = 4x - 2$. Find the volume of the solid under the graph of $f$ over the region $S$.