SQ18

a. From the experimental data obtained from NIST (Note AP-IV-(38)),

the energy level of the following transitions :

	(cm^{-1})	(eV)
$2p, j=\frac{3}{2} \to 1s$	82259.2850	10.198506
$2p, j=\frac{1}{2} \to 1s$	82258.9191	10.198460

The difference in eV is about 4.6×10^{-5} eV.

Imagine that the difference is due to the alignment / anti-alignment with the internal field B_{int}, ie, the correction is $\pm \mu_B B_{int}$.

We have the following :

$$2\mu_B B_{int} = 4.6 \times 10^{-5} \text{ eV}$$

Put the value of Bohr magneton μ_B to be (Note AP-II-(2)),

$$\mu_B = 5.79 \times 10^{-5} \text{ eV / Tesla}$$

$$\Rightarrow \quad B_{int} \approx 0.397 \text{ Tesla}$$

For the fact that we usually have only a few Tesla in the lab. magnetic field, B_{int} is not small!

b. For the sodium transitions from 3p to 3s state,

the energy levels are :

	(nm)	(eV)
$3p, j=\frac{3}{2} \to 3s$:	588.592	2.10645
$3p, j=\frac{1}{2} \to 3s$:	588.995	2.10501

Set the difference to be $2\mu_B B_{int}$, we can obtain :

$2\mu_B B_{int} = 1.44 \times 10^{-3}$ eV , $B_{int} = 12.44$ Tesla, (Very Big !!)

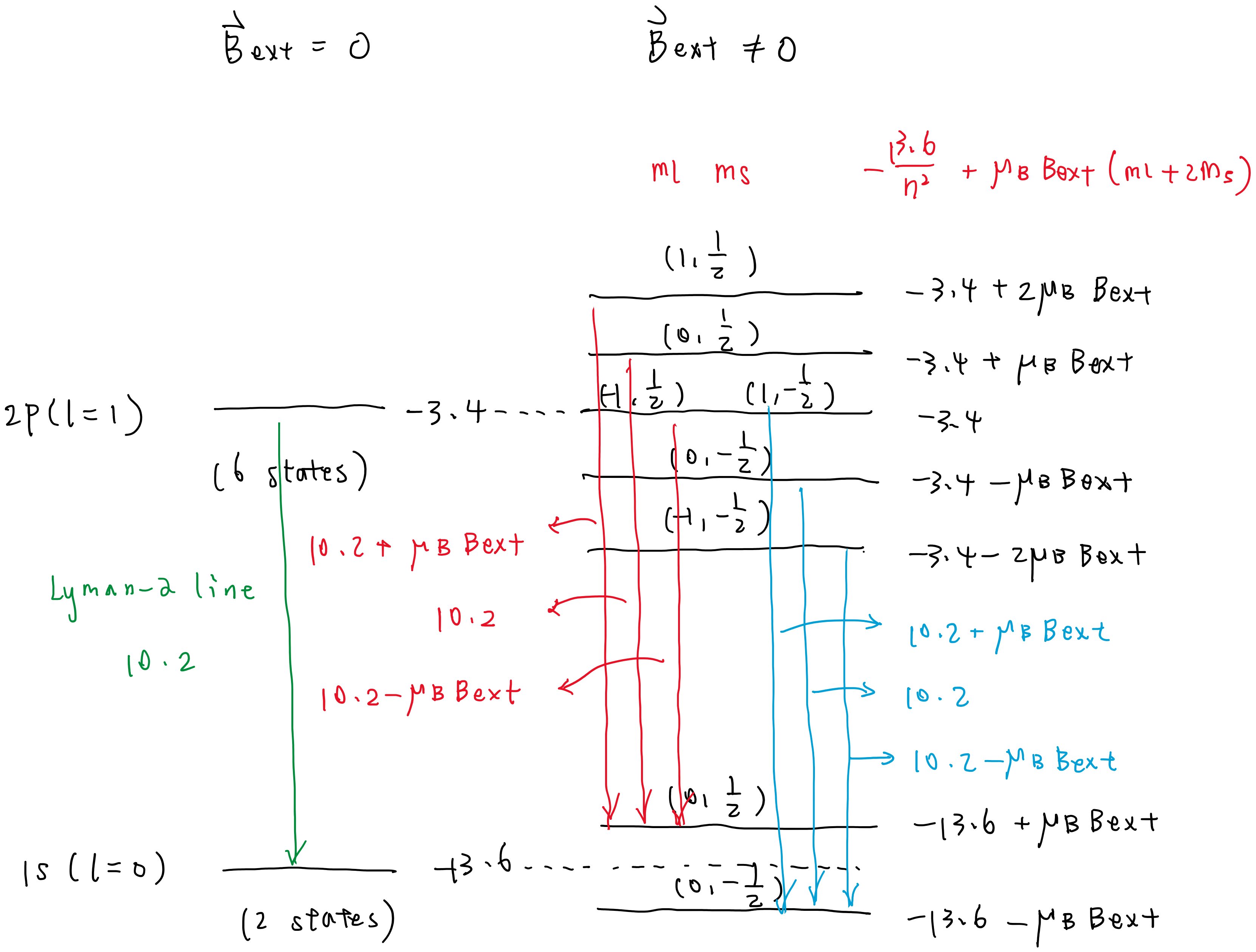
SQ 19.

We recall in the strong-field Zeeman effect,

$$E_{n l m_l m_s} \approx -\frac{13.6}{n^2} + \mu_B B_{ext}(m_l + 2m_s),$$

where the spin-orbit interaction term $m_l m_s \hbar^2 \langle f(r) \rangle$ is ignored.

We consider the Lyman $2p \to 1s$ transition for H-atom:



By the selection rule $\Delta m_s = 0$, $\Delta m_l = 0, \pm 1$,

we find that the Lyman α-line (energy difference is 10.2) splits into 3 lines (energy differences are $10.2 + \mu_B B_{ext}$, 10.2, $10.2 - \mu_B B_{ext}$ respectively).

SQ20

Physical Quantities	Atomic Unit is measured in terms of	SI Equivalent
Mass	m_e	9.1094×10^{-31} kg
Charge	e	1.6022×10^{-19} C
Distance	$a_0 = \dfrac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$	5.2918×10^{-11} m
Energy	$E_h = \dfrac{m_e e^4}{16\pi^2\varepsilon_0^2\hbar^2} = \dfrac{e^2}{4\pi\varepsilon_0 a_0}$	4.3597×10^{-18} J 27.2114 eV
Angular momentum	$\hbar$	1.0546×10^{-34} Js
Permittivity	$4\pi\varepsilon_0$	1.1127×10^{-10} $C^2 J^{-1} m^{-1}$

SQ20

$$\hat{H}_{(SI)} = -\frac{\hbar^2}{2m_e}\nabla^2_{\vec{r}} - \frac{e^2}{4\pi\varepsilon_0 r}$$

$$= \frac{e^2}{4\pi\varepsilon_0 a_0}\left[-\frac{\hbar^2}{2m_e}\frac{4\pi\varepsilon_0 a_0}{e^2}\nabla^2_{\vec{r}} - \frac{a_0}{r}\right]$$

$$= E_h\left[-\frac{\hbar^2}{2m_e}\frac{4\pi\varepsilon_0}{e^2 a_0}a_0^2\nabla^2_{\vec{r}} - \frac{a_0}{r}\right]$$

Define $\vec{r}\,' = \frac{\vec{r}}{a_0} = \left(\frac{x}{a_0}, \frac{y}{a_0}, \frac{z}{a_0}\right) = (x', y', z')$

$$\nabla^2_{\vec{r}\,'} = \frac{\partial^2}{\partial x'^2} + \frac{\partial^2}{\partial y'^2} + \frac{\partial^2}{\partial z'^2} = a_0^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) = a_0^2\nabla^2_{\vec{r}}$$

$$\therefore \hat{H}_{(SI)} = E_h\left[-\frac{\hbar^2}{2m_e}\frac{4\pi\varepsilon_0}{e^2 a_0}\nabla^2_{\vec{r}\,'} - \frac{1}{r'}\right]$$

$$\frac{\hat{H}_{(SI)}}{E_h} = -\frac{\hbar^2}{2m_e}\frac{4\pi\varepsilon_0}{e^2 a_0}\nabla^2_{\vec{r}\,'} - \frac{1}{r'}$$

Since $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{m_e e^2}$, $\frac{\hbar^2 4\pi\varepsilon_0}{m_e e^2 a_0} = 1$. It follows that

$$\hat{H}_{(atomic)} = \frac{\hat{H}_{(SI)}}{E_h} = -\frac{1}{2}\nabla^2_{\vec{r}\,'} - \frac{1}{r'} = -\frac{1}{2}\nabla^2_{\vec{r}} - \frac{1}{r}$$