PHYS3022 APPLIED QUANTUM MECHANICS

SAMPLE QUESTIONS FOR DISCUSSION IN WEEK 7 EXERCISE CLASSES (9 - 13 March 2020)

What are Sample Questions (SQs)? TA will discuss the **SAMPLE QUESTIONS** in exercise classes. The Sample Questions are designed to serve several purposes. They either review what you have learnt in previous courses, supplement our discussions in lectures, or closed related to the questions in an upcoming Problem. You are encouraged to think about (or work out) the sample questions before attending exercise class and ask the TA questions.

SQ15 - Normal Zeeman Effect: $P \to D$ transitions allowed by $\Delta m_L = 0, \pm 1$
SQ16 - The rule of adding two angular momenta gives the right number of states
SQ17 - The "minimal substitution rule" in generating the $-\vec{\mu}_L \cdot \vec{B}$ term in Hamiltonian and more

SQ15 *Normal Zeeman Effect: $P \to D$ transitions allowed by $\Delta m_L = 0, \pm 1$*

Background: This SQ is about the normal Zeeman effect. The notations here look unusual, as capital letters are used. There is a physical reason. Hydrogen has only one electron and its spin angular momentum cannot be zero. We ignored effects of spin in the normal Zeeman effect. Therefore, the effect (one spectral line becomes 3 lines) is not observed in hydrogen. It is observed in atoms where spin effects can be ignored. One such case is the helium atom, where the spin angular momenta of the two electrons could add up to give a zero total spin angular momentum. For atoms other than hydrogen (and hydrogen-like atoms), we use capital letters to represent quantities of the **whole atom**. So L is the orbital angular momentum of the atom after adding up the contributions of orbital angular momenta from each electron. Thus, P means $L = 1$ with $m_L = 1, 0, -1$ and so on.

Consider the possible transitions from an upper P to a lower D level in a static magnetic field. **Show that** the allowed transitions give two more spectral lines one on each side of the line in the zero field case, according to the selection rules of $\Delta m_L = 0, \pm 1$.

SQ16 *The rule of adding two angular momenta gives the right number of states*

In class, we illustrated that for p states ($\ell = 1$) including spin ($s = 1/2$), there are 6 states. Changing the descriptions to $j = 3/2$ [4 states behind m_j] and $j = 1/2$ [2 states behind m_j], there are also 6 states – no more and no less!

It is unnecessary for one of the angular momenta quantum numbers to be $1/2$. In general, we could add an angular momentum of quantum number j_1 and another angular momentum of quantum number j_2 to form a total angular momentum for which the quantum number j takes on a range of values determined by j_1 and j_2. While deriving the rule of adding two angular momenta in QM is beyond the scope of our course, applying the rules helps understand features in systems consisting of many electrons such as atoms and molecules. We will make use of the results later. The rule is that given j_1 and j_2, the total angular momentum quantum number j can go from $j_1 + j_2$ to $|j_1 - j_2|$ in steps of 1.

TA: **Show that** counting the states by the labels $(j_1, m_{j_1}, j_2, m_{j_2})$ and by the labels (j_1, j_2, j, m_j) account for the **same number of states**.

TA: **Illustrate** this general statement by considering $j_1 = 5/2$ and $j_2 = 1$. **Show** that the two labelling schemes account for the same number of states.

SQ17 *The "minimal substitution rule" in generating the $-\vec{\mu}_L \cdot \vec{B}$ term in Hamiltonian and more*

Background: When an external magnetic field is applied to an atom, a magnetic interaction energy coming from the interaction between the magnetic dipole moment $\vec{\mu}_L$ accompanying the

orbital angular momentum $\vec{L}$ of the electron and the applied magnetic field $\vec{B}$ (or $\vec{B}_{ext}$ to be explicit) is added to the Hamiltonian. The interaction energy is given by $-\vec{\mu}_L \cdot \vec{B}$. In QM, this term becomes an operator and it is added to the Hamiltonian. The $\vec{\mu}_L$ becomes $\hat{\vec{\mu}}_L$. A consequence is the normal Zeeman effect. **Note that spin angular momentum is ignored here.** All is fine. This follows from the "think classical" and then "go quantum" procedure.

We ask a more general question: "Is there a standard procedure/recipe to incorporate the effect of an applied $\vec{B}$ field in QM?" The answer is yes and it is a big yes! Here, we introduce **a standard procedure** in QM to include the effect of $\vec{B}$ and to generate the $-\vec{\mu}_L \cdot \vec{B}$ term **automatically**. The procedure is applicable to many other occasions, e.g. incorporating the effects of EM fields on an electron quantum mechanically into the Dirac equation and in quantum field theories.

In QM, the **vector potential** $\vec{A}$ plays a more important role than the magnetic field $\vec{B}$. Recall that $\vec{B} = \nabla \times \vec{A}$. In QM, when an applied magnetic field acts on a charged particle of charge q, the effect is captured by (i) writing down the Hamiltonian *without* the magnetic field effect; and (ii) **replacing the linear momentum** $\vec{p}$ in the Hamiltonian **by** $\vec{p} - q\vec{A}$, i.e., making the substitution $\vec{p} \to \vec{p} - q\vec{A}$, where $\vec{A}$ is the vector potential that can generate the field $\vec{B}$. When we apply the procedure to the electron in a hydrogen atom in the presence of an applied field $\vec{B} = B\hat{z}$, we have

$$\hat{H} = \frac{(\vec{p} + e\vec{A})^2}{2m} + V(r) \tag{1}$$

where $V(r) = -e^2/(4\pi\epsilon_0 r)$ is the Coulomb potential energy term for a hydrogen atom, and $V(r)$ is (assumed to be) a spherically symmetrical potential for other atoms.

TA: For $\vec{B} = B\hat{z}$ (no loss of generality), **choose** a proper $\vec{A}$ that works and **show** that a term of the form $-\vec{\mu}_L \cdot \vec{B}$ emerges (without *a priori* knowing there is an orbital magnetic dipole moment)! Identify $\vec{\mu}_L$. Also **show** that **an extra term** of the order A^2 (thus B^2) also emerges as a by-product.

Physics remarks: (a) Physically, the term $-\vec{\mu}_L \cdot \vec{B}$ is a **paramagnetic** response, as it prefers the alignment of the magnetic dipole moment with $\vec{B}$. The extra $\sim A^2$ term is a **diamagnetic response** of the orbiting electron. It can be treated by the 1st order perturbation theory. It is analogous to the Lenz law. The response in a loop threaded through by a changing magnetic field is a current that opposes the change. Since the magnetic field is typically not big, the paramagnetic term is more important than the diamagnetic term. In some cases where $L = 0$ (so $\vec{\mu}_L = 0$), the diamagnetic term becomes important. (b) If we have a scalar potential ϕ in addition to $\vec{A}$, you may immediately think that there should be a term $q\phi$ added to the energy (Hamiltonian). You are right! (c) The magical thing is that the substitution **generates how a charged particle interacts with an external field**. For EM interaction, the Maxwell's equations and the Lorentz force govern the behavior of EM fields and how a charge interacts with EM fields. Thus the substitution rule gives nothing new. It simply confirms what is known. However, for the cases where the form of the interaction term is not clearly known (e.g. other interactions in particle physics), this substitution serves as a guiding principle in key developments in quantum (gauge) field theories.