PHYS3022 APPLIED QUANTUM MECHANICS

SAMPLE QUESTIONS FOR DISCUSSION IN WEEK 4 EXERCISE CLASSES (17-21 February 2020) via online learning

What are Sample Questions (SQs)? The Sample Questions are designed to serve several purposes. They either review what you have learnt in previous courses, supplement our discussions in lectures, or closed related to the questions in an upcoming Problem. You are encouraged to think about (or work out) the sample questions before attending exercise class and ask the TA questions.

SQ9: Postulates of Quantum Mechanics (see class notes and clips in course web page)
SQ10: Using $(\vec{\sigma}\cdot\vec{p})^2$ to treat $(p_x^2+p_y^2+p_z^2)$ for any vector $\vec{p}$ and its implications - Problem 2.7

SQ9 *Postulates of Quantum Mechanics*

Students should read the Class Notes on Postulates of Quantum Mechanics. It is a short summary as a few postulates on what we learned from examples in QM I and the first part of Applied QM. I cited the postulates of QM as given in three standard textbooks. They just use different words to say the same things. Basically, they include: state or wavefunction, operators for physical quantities, eigenvalues of an operator give the possible measurement outcomes of a quantity, how expectation value is calculated, time evolution is governed by the time- dependent Schrödinger equation. This part will not be discussed in class. There is a link to a 12-minute discussion on this part in the course webpage. This SQ will give a brief discussion on these postulates.

SQ10 *Using $(\vec{\sigma}\cdot\vec{p})^2$ to treat $(p_x^2+p_y^2+p_z^2)$ for any vector $\vec{p}$ and its implications - Problem 2.7*

We will discuss the solutions to Problem 2.7 and its implications. Below is Problem 2.7 in Problem Set 2.

When you were in primary school, you knew that $p_x^2-p_y^2=(p_x+p_y)(p_x-p_y)$. When you were in secondary school, you knew that $p_x^2+p_y^2=(p_x+ip_y)(p_x-ip_y)$. It was a big step forward with the non-trivial idea of i. Now you are studying physics in university, what's next? Naturally, the question is to simplify $p_x^2+p_y^2+p_z^2$, which is the magnitude squared of a 3-component vector $\vec{p}=p_x\hat{i}+p_y\hat{j}+p_z\hat{k}$. A related question is whether we can write $E^2-c^2(p_x^2+p_y^2+p_z^2)$ into a product of two terms.

Let's form a vector $\vec{\sigma}=\sigma_x\hat{i}+\sigma_y\hat{j}+\sigma_z\hat{k}$, where σ_i are the Pauli matrices. That is to say, each component is a 2×2 matrix. Following what you know about dot product of two vectors and matrix manipulations, **evaluate** $\vec{\sigma}\cdot\vec{p}$ and $(\vec{\sigma}\cdot\vec{p})^2$. [Reamrk: Inspect and appreciate the result and the beauty of mathematics. Now at the university level, even i is not sufficient, you need to use matrices.]

Hence, **express** $E^2-c^2(p_x^2+p_y^2+p_z^2)$ as a product of two factors. [Note the form is highly non-trivial as we must invoke 2×2 matrices to do it. It is not just writing some math in fancy form. It is a necessity.]

[Remarks: You may recognize $E^2-c^2(p_x^2+p_y^2+p_z^2)=0$ as a relativistic relation between energy and the momentum for a massless particle (no m^2c^4 term), if $\vec{p}$ is taken to be the momentum. Hence, you showed that $(E+something)(E-something)=0$, and that something is a matrix. In going quantum, we would get $(E-something)\psi=0$, which is a relativistic QM equation. In fact, it is the Dirac equation for massless fermions. It sounds useless. Not so! It is now a popular topic due to the fact that electrons in graphene (one layer of graphite) behave like massless fermions! The equation is more complicated for massive fermions. We need to invoke 4×4 matrices called the Dirac matrices, which are formed by stacking up the Pauli matrices. The resulting equation is the famous Dirac equation.]