Is the Universe Rotating?

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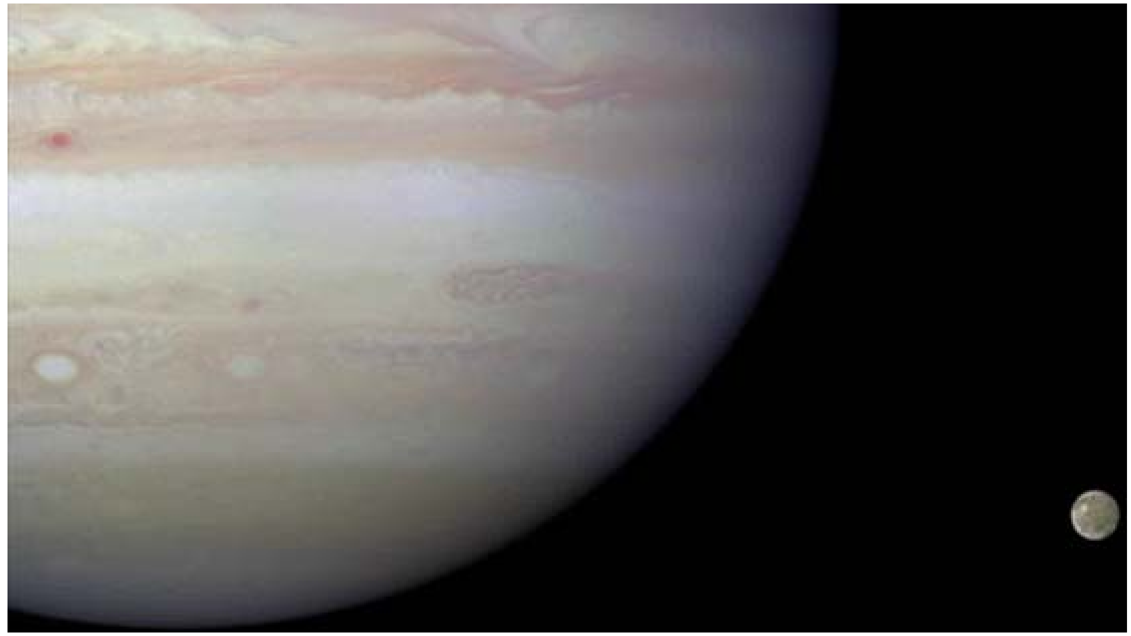
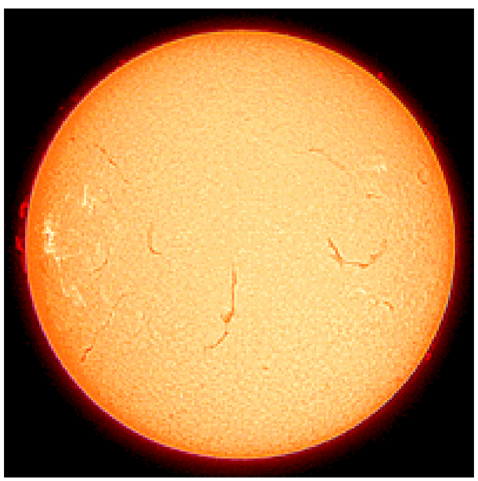
Toutatis' spin axis traces a curve around the asteroid's surface once every 5.41 days

Animation courtesy NASA



Jupiter rotates once every ~ 10hrs.

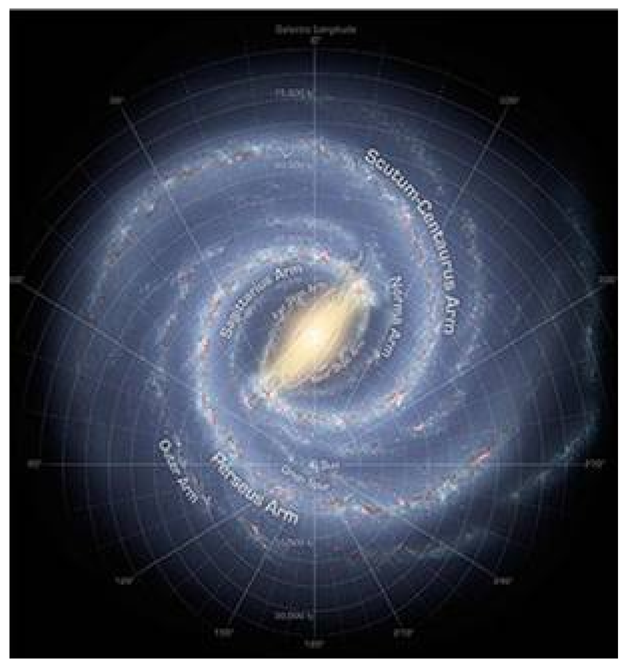
Animation courtesy NASA



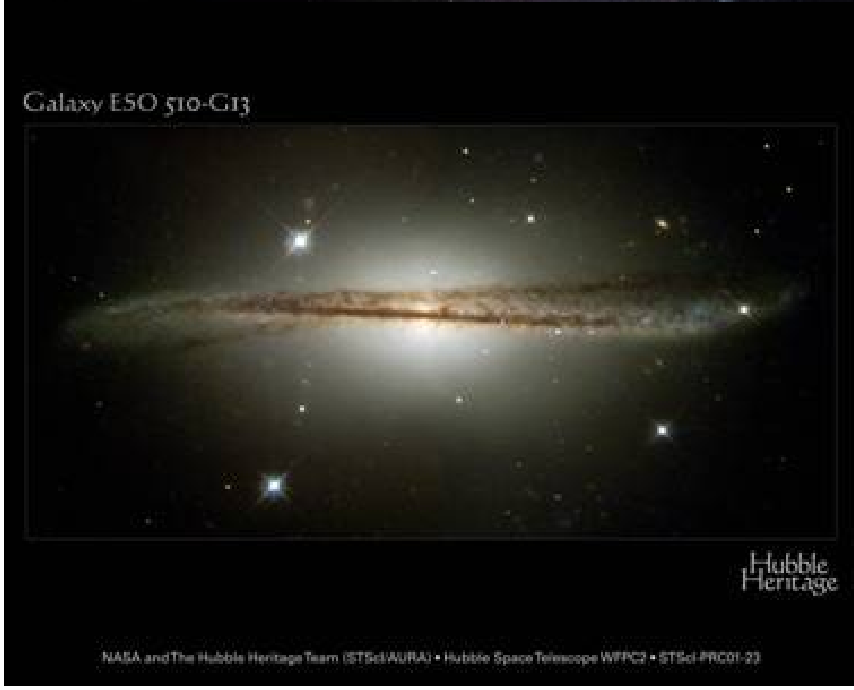
Sun rotates once every ~25 days

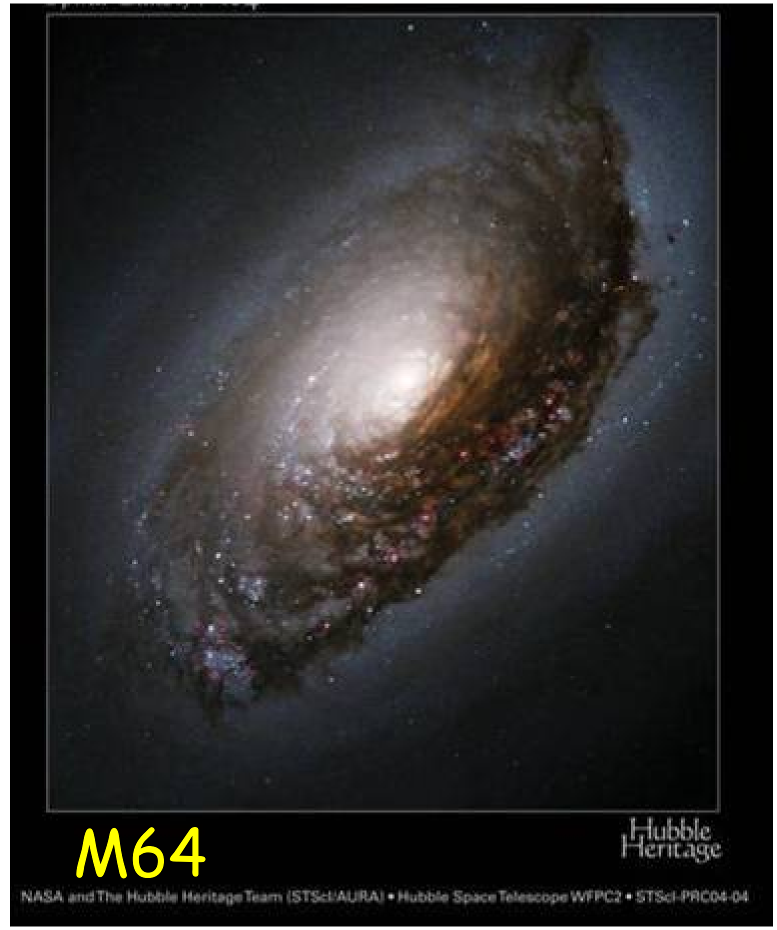
Animation by mc^2

Everything spins!



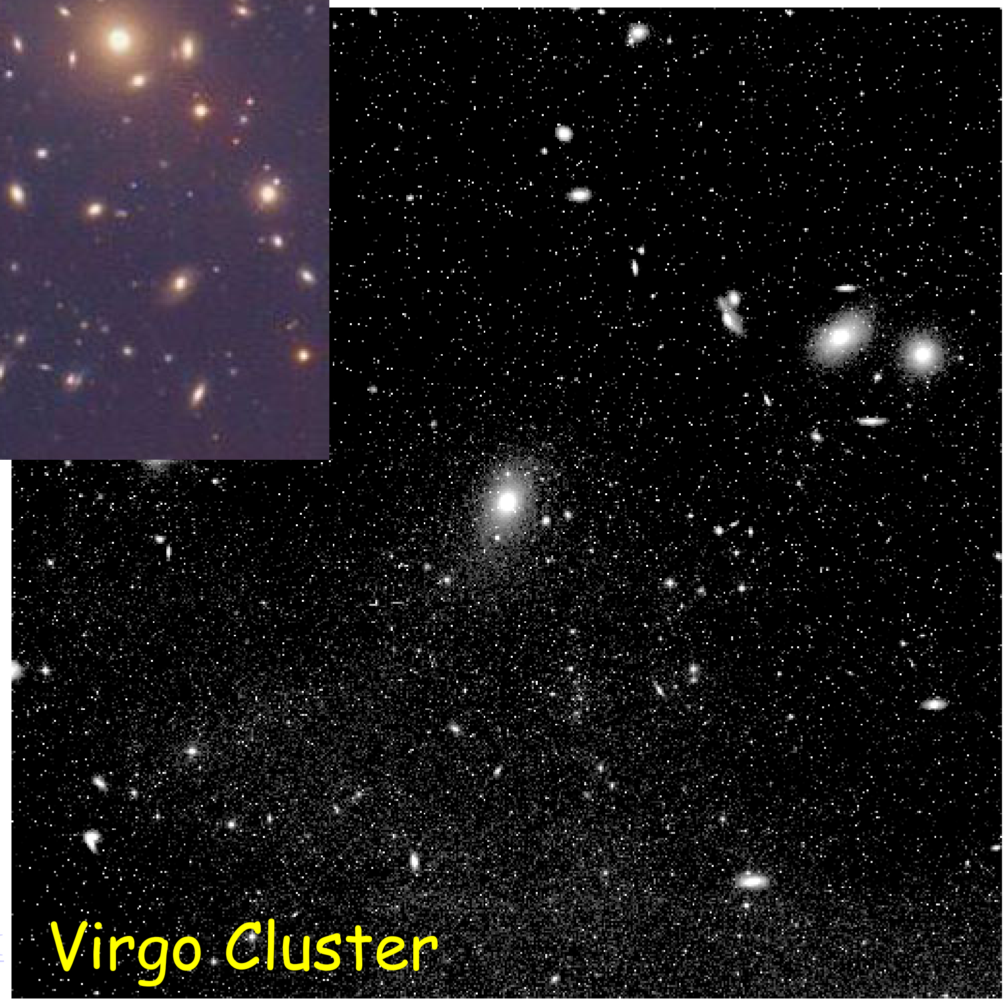
Milky Way is rotating, with period ~ 220 MYr at Sun's orbit





Images courtesy NASA/STScI

http://antwrp.gsfc.nasa.gov/apod/ap020203.html



Even clusters of galaxies rotate → discovery of dark matter

Photo credit: Digitized Sky Survey, Palomar Observatory, STScI

Is the Universe Rotating?

I. Some history: Plato, Copernicus, Newton, Gamow, Gödel, Li, Barrow, Hawking

II. Recent controversy: AOE – is there a preferred direction in the universe?

III. Our own work: constraining the rotational speed of the universe

Summary

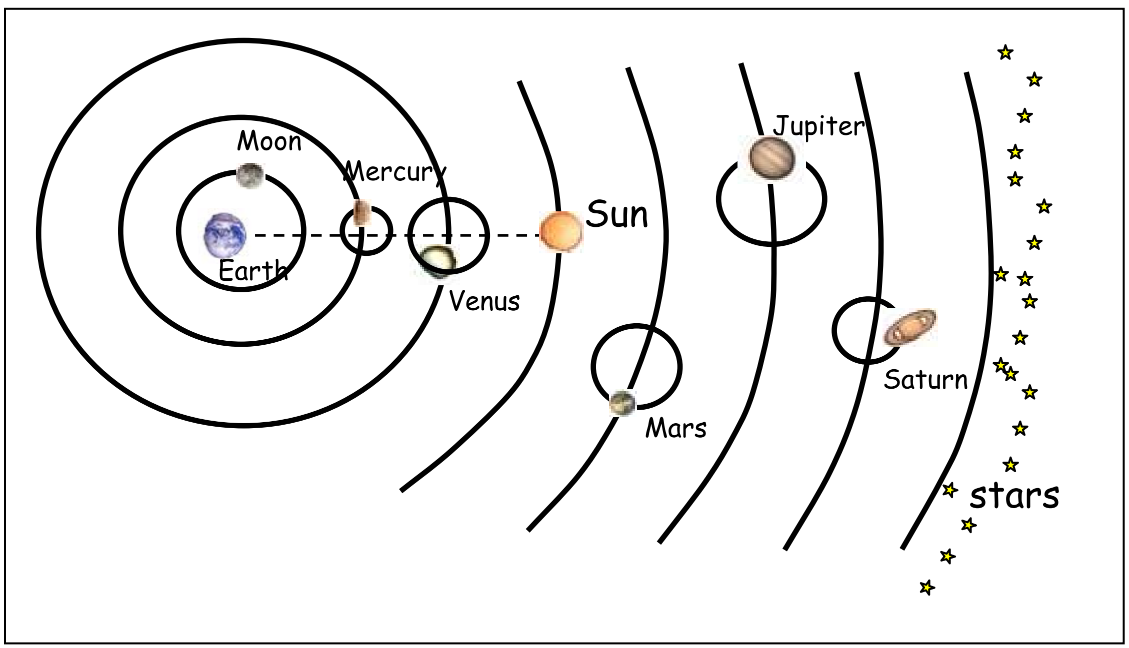
I. If the universe rotates:

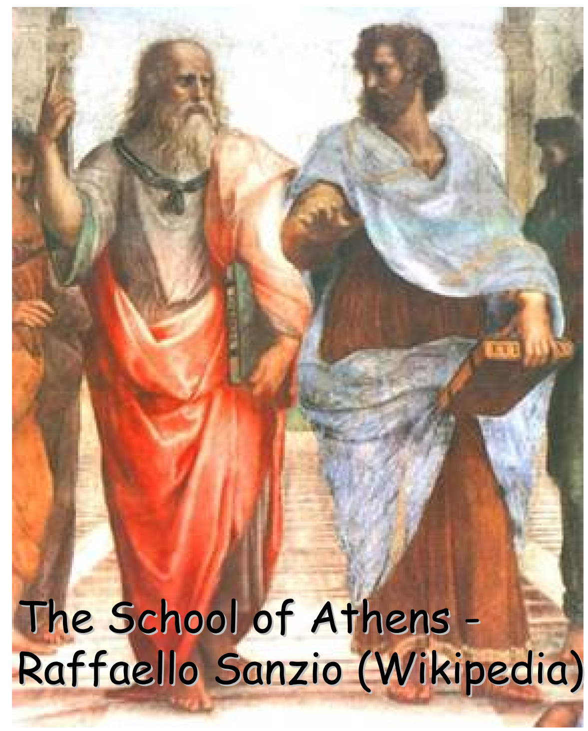
- May explain the rotation of most stellar objects
- But may violate causality
- May impact our understanding of time (time travel possible?)
- May need to throw away inflation theory, revise standard cosmological theory

II. Using data to constrain rotation speed: highly dependent on model, not conclusive yet

Plato-Aristotle Cosmology

- Plato: heavenly objects only take up perfect motion = circular motion with uniform speed
- Aristotle: Earth sits still at the center of the universe → a finite, rotating universe; human is unique



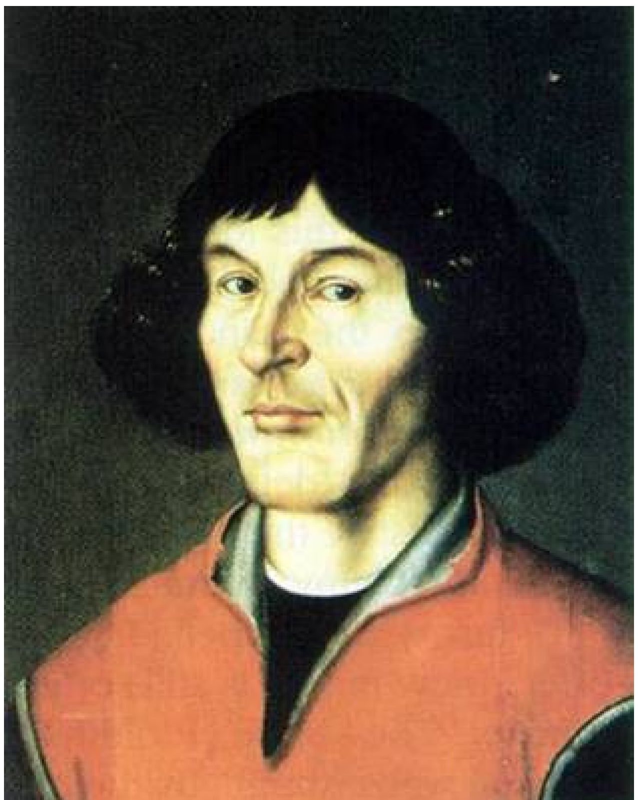


Looking North: star trails, rotation of the celestial sphere

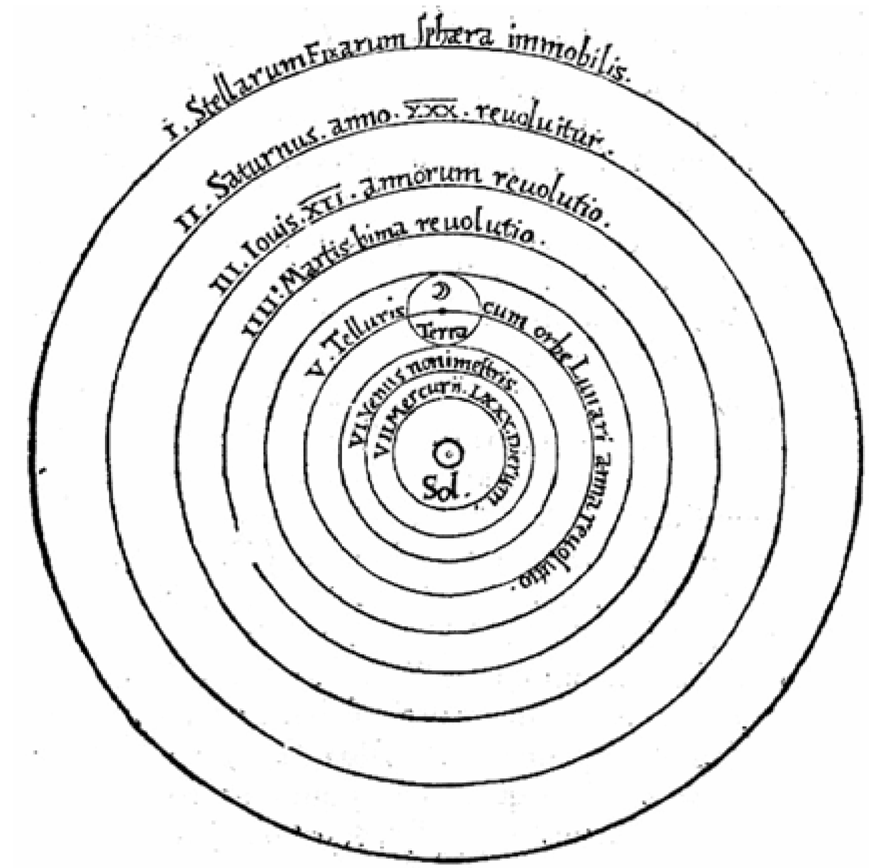
Copernicus' Cosmology (simplified version)

- More economical to have the earth rotating
- Stars and the sun are fixed; Earth and other planets orbit around the sun

Revolution!

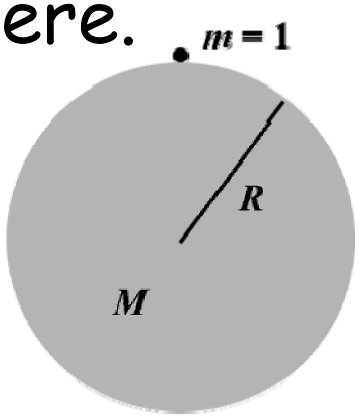


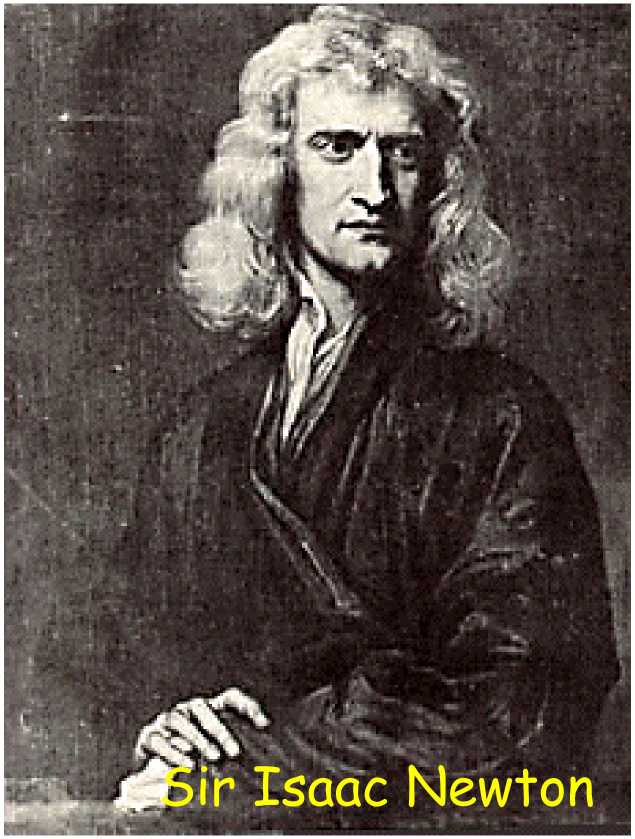
Nicolaus Copernicus



Newtonian Cosmology

- Space is absolute and static
- Unstable if the universe is finite: forces not balanced anywhere except at the exact center, but unstable equilibrium even there.





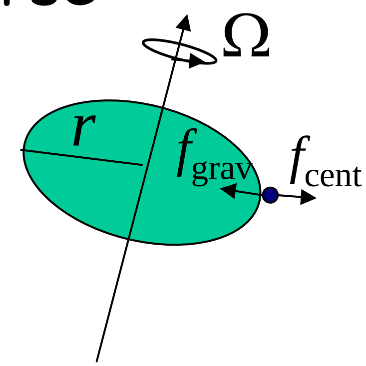
Newton: the universe must be infinite and static! $\rightarrow$ no rotation!

Rotation to the rescue (partially)!

Stockum (1937), Gödel (1949):

Only matter rotates, space remains static

Uniform density, rigidly rotating Newtonian universe



$$\nabla^2\Phi = 4\pi G\rho, \quad \rightarrow \quad \Phi = \pi G\rho(x^2 + y^2) = \pi G\rho r^2,$$

$\therefore$ Gravitational force towards rotation axis: $\vec{f}_{\mathrm{grav}} = -\nabla\Phi = -2\pi G\rho\vec{r},$

Centrifugal force: $\vec{f}_{\mathrm{cent}} = \Omega^2\vec{r},$

$\Omega^2 = 2\pi G\rho$ (both constant) $\rightarrow$ universe static everywhere!

Coriolis force = $\vec{f}_{\mathrm{Cor}} = 2\vec{v} \times \vec{\Omega}$ same everywhere

Homogeneity, but anisotropic

W. J. van Stockum, Proc. R. Soc. Edinburgh **57**, 135 (1937).

W. Rindler, AJP **77**, 498 (2009).

Cosmic Microwave Background

Origin of elements

Alpha decay

Scanned at the American Institute of Physics

- rotation of stellar objects caused by rotation of the universe
- Rotation of the universe can be observed
- A solution of GR corresponding to rotating universe can probably be constructed.

http://www.aip.org/history/ohilist/4325.html

LETTERS TO THE EDITORS

The Editors do not hold themselves responsible for opinions expressed by their correspondents. No notice is taken of anonymous communications

G. Gamow, Nature **158**, 549 (1946).

Rotating Universe ?

ONE of the most mysterious results of the astronomical studies of the universe lies in the fact that all successive degrees of accumulation of matter, such as planets, stars and galaxies, are found in the state of more or less rapid axial rotation. In various cosmogonical theories the rotation of planets has been explained as resulting from the rotation of stars from which they were formed. The rotation of stars themselves (in particular that of *B*-stars) can be presumably reduced to their origin from the rotating gas-masses which form the spiral arms of various galaxies. But what is the origin of galactic rotation ?

If, according to the current theories, we consider the galaxies as the result of gravitational instability of the originally uniform distribution of matter in space, we will find it very difficult to understand why such condensations are in most cases found in the state of rather fast rotation. In fact, on the basis of statistical distribution of angular momentum, we would rather expect such condensations to show no more rotation than the water droplets in a fog formed from over-saturated vapour. Barring the possible explanation of the rotation of galaxies on the basis of the alleged irregular turbulent motion of the masses of the universe, we can ask ourselves whether it is not possible to assume that *all matter in the visible universe is in a state of general rotation around some centre located far beyond the reach of our telescopes ?*

The answer to such, at first sight fantastic, question need not wait until much larger telescopes shall have been built. It can be, in fact, settled by present means of observation. We know that the rotation of the stars of our system around the galactic centre can be proved by the study of the so-called Oort-effect in the radial velocities of comparatively near stars. In fact, due to the phenomenon of differential rotation, the mean radial velocities of stars located along the galactic plane show a double-sine periodicity with nodal axes directed parallel and perpendicular to the line connecting the sun with the centre of rotation. Thus if the realm of galaxies as seen through Mt. Wilson telescope represents only a small part of a much larger system (a 'super-galaxy' in the super-Shapley sense) rotating around a distant centre, careful observations of mean radial velocities of galaxies located in different regions of the sky should reveal similar periodicity.

The existence of this effect would prove general rotation of the universe and indicate the direction towards the rotation centre without, however, giving us its distance. Thus, it seems that the answer to the problem of universal rotation lies within the grasp of modern astronomical technique.

It must be added in conclusion that in the language of the general theory of relativity such a rotating universe can be probably represented by the group of anisotropic solutions of the fundamental equations of cosmology.

G. GAMOW

Department of Physics.

Kurt Gödel

Gödel and Einstein at Institute for Advanced Study, Princeton. Photo from IAS Archives.

Gödel's Incomplete Theorems (1930)

http://en.wikipedia.org/wiki/G%C3%B6del%27s_incompleteness_theorems

'In any formal system adequate for number theory there exists an undecidable formula - that is, a formula that is not provable and whose negation is not provable.'

'the consistency of a formal system adequate for number theory cannot be proved within the system.'

- R. Goldstein, 'Incompleteness', W. W. Norton, 2005.

Fundamental limitations of formal logic systems (cf. Uncertainty Principles, Relativity)

Einstein 'came to the Institute merely in order to have the privilege of walking home with Gödel.' – O. Morgenstern

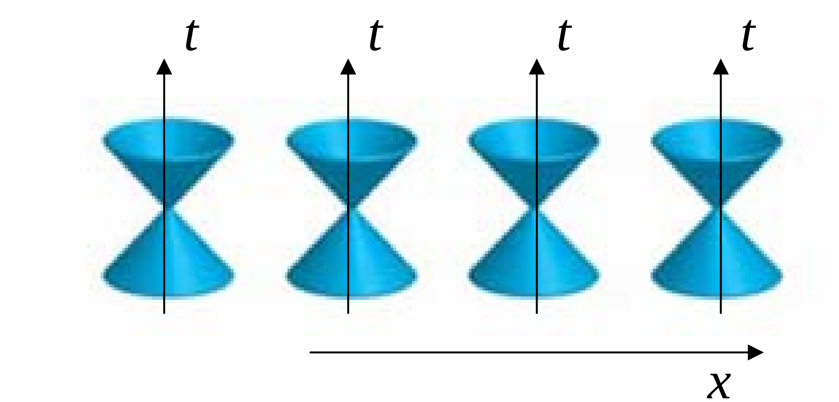
http://discovermagazine.com/2002/mar/featgodel

Gödel's Rotating, Static Universe (1949)

$$ds^2 = a^2[-d\tau^2 + dx^2 - (e^{2x}/2)dy^2 + dz^2 - \boxed{2e^x d\tau dy}]$$

There exists closed time-loops: after completing an orbit, one goes back to the starting point at an earlier time!

Light cones in standard cosmology



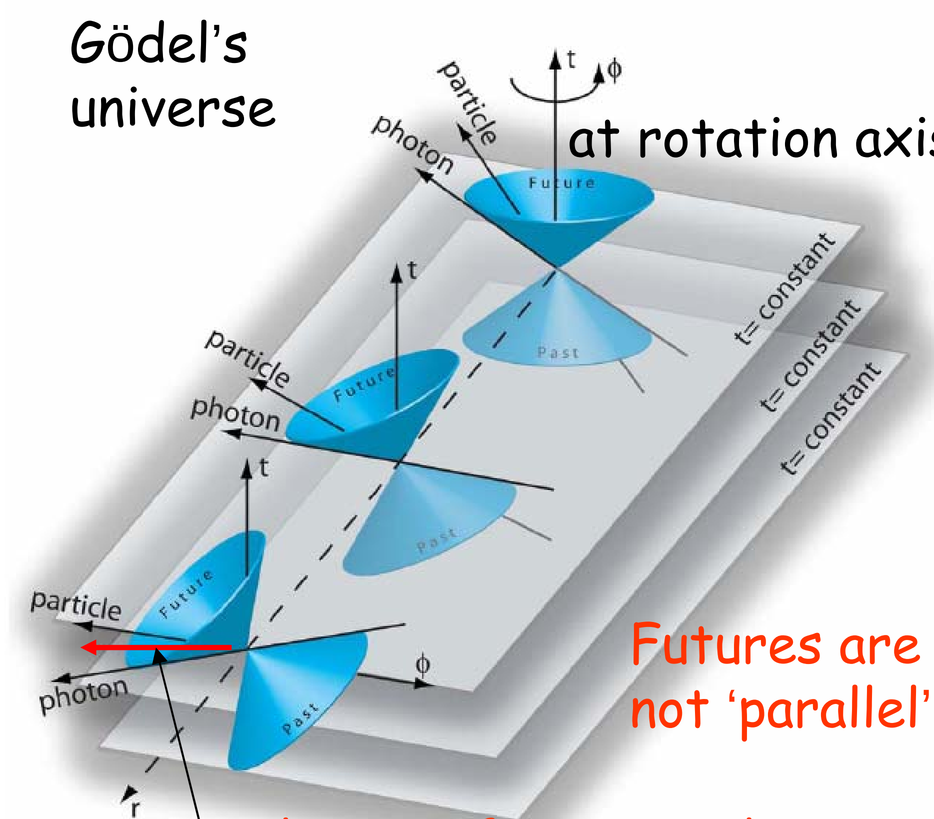
A universal time with future all pointing in the direction of decreasing density/temperature

K. Gödel, Rev. Mod. Phys. **21**: 447–450 (1949).

W. Rindler, Am. J. Phys. **77**, 498 (2009).

http://en.wikipedia.org/wiki/G%C3%B6del_metric

Gödel's universe



at rotation axis

Futures are not 'parallel'

At large r, fast particles go backward in time

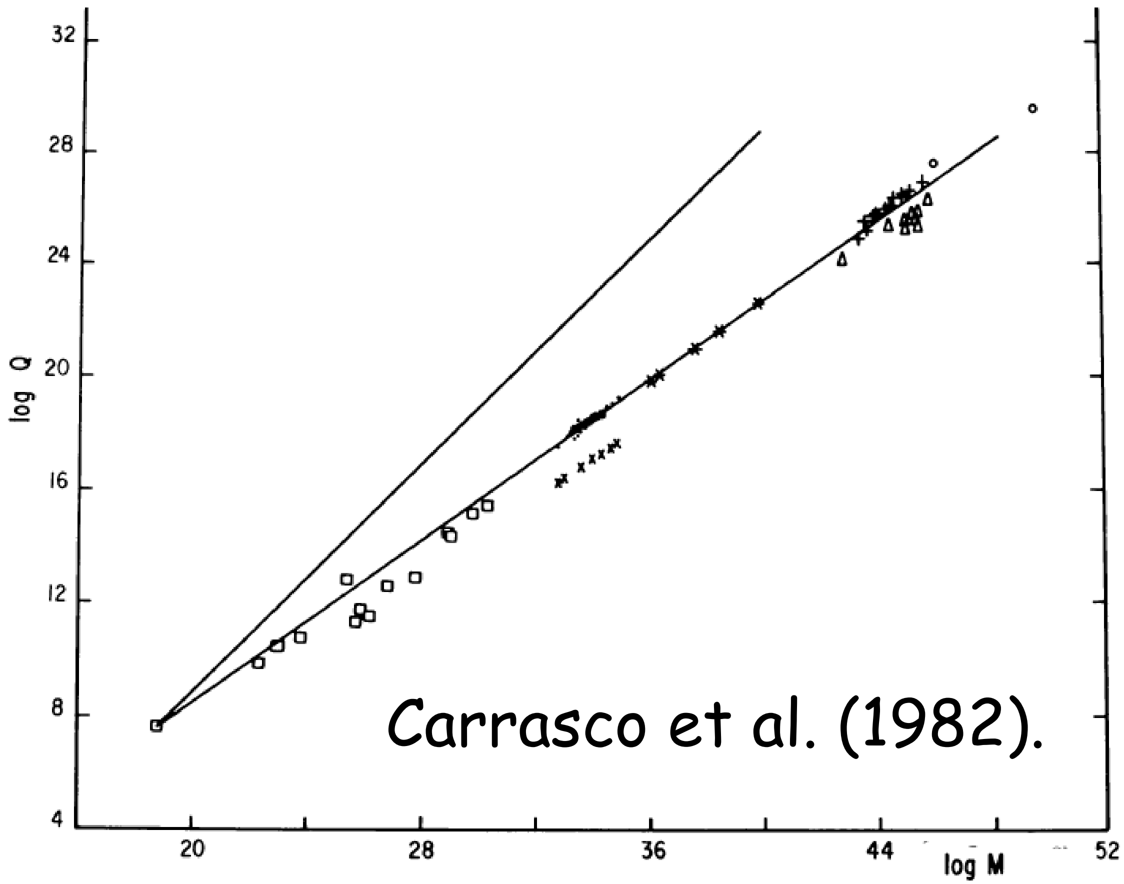
Gödel's Rotating, Static Universe (1949)

- Gödel's birthday gift to Einstein
- Exact solution of GR, motivated by Gamow's paper
- Violates causality! One can go back to kill one's father/mother!
- Einstein's response: Nature must have ways to prevent Gödel's space-time from materializing
- Static → considered to be just a toy model, largely ignored
- Modern view of physics: if it's not forbidden by some principles, it should be there! Could there be many Gödel universes? Ours is just an accident?
- Time is just an illusion?

P. Yourgrau, 'A World Without Time: The Forgotten Legacy Of Gödel And Einstein'

But Gamow's conjecture is still lurking around …

http://www.aip.org/history/ohilist/4325.html



Empirical relation between angular momentum (J) and mass of galaxies (M):

$$J = kM^{5/3}$$

k ~ 0.4 (cgs units)

valid also for many other celestial objects!

Class of objects	$j \propto M^{\alpha}$		$Q \propto M^{\beta}$ Corrected for density effect		N
	α	Corr. coeff.	β	Corr. coeff.	
Asteroids, satellites, and planets	0.66 ± 0.06	0.955	0.66 ± 0.04	0.976	18
Contact binaries	0.71 ± 0.05	0.947	0.76 ± 0.02	0.992	29
Visual binaries	0.67 ± 0.07	0.711	0.68 ± 0.03	0.908	99
Clusters, bulges, and elliptical galaxies	0.66 ± 0.07	0.998	0.62 ± 0.01	0.996	18
Spiral galaxies[a]	0.72 ± 0.05	0.93	—	—	88
Spiral galaxies and super cluster	0.84 ± 0.04	0.975	0.74 ± 0.03	0.987	25
All objects	0.94 ± 0.09	0.987	0.71 ± 0.01	0.994	190

[a] Vettolani et al. (1980)

Taken from L. Carrasco et al., Astron. Astrophys. **106**, 89 (1982).

Effect of the Global Rotation of the Universe on the Formation of Galaxies

General Relativity and Gravitation **30**, 497 (1998).

Li-Xin Li[1,2]

Received July 1, 1997. Rev. version October 1, 1997

The effect of the global rotation of the universe on the formation of galaxies is investigated. It is found that the global rotation provides a natural origin for the rotation of galaxies, and the morphology of the objects formed from gravitational instability in a rotating and expanding universe depends on the amplitude of the density fluctuation, different values of the amplitude of the fluctuation lead to the formation of elliptical galaxies, spiral galaxies, and walls. The global rotation gives a natural explanation of the empirical relation between the angular momentum and mass of galaxies: $J \propto M^{5/3}$. The present angular velocity of the universe is estimated at $\sim 10^{-13}$ rad yr^{-1}.

Physics: Coriolis force + gravitational collapse + expanding, rotating background

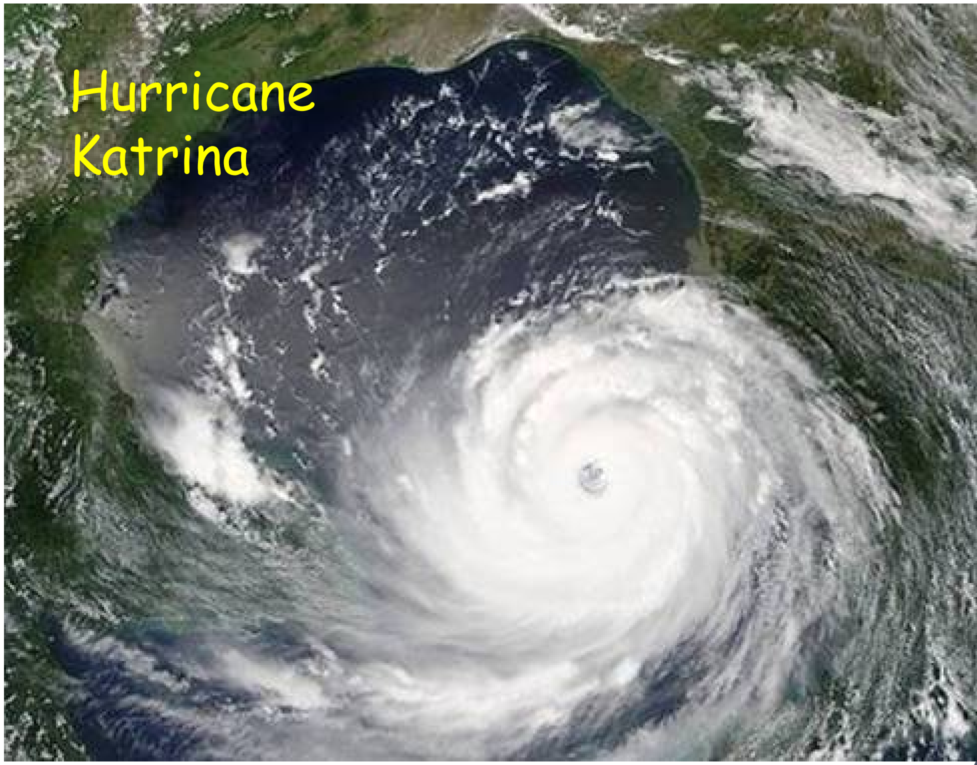
Similar to formation of typhoons!

- Derived $J \propto M^{5/3}$

based on formation of galaxies in a rotating universe, and conservation of angular momentum.

- Explained two types of galaxies (spiral, elliptical).

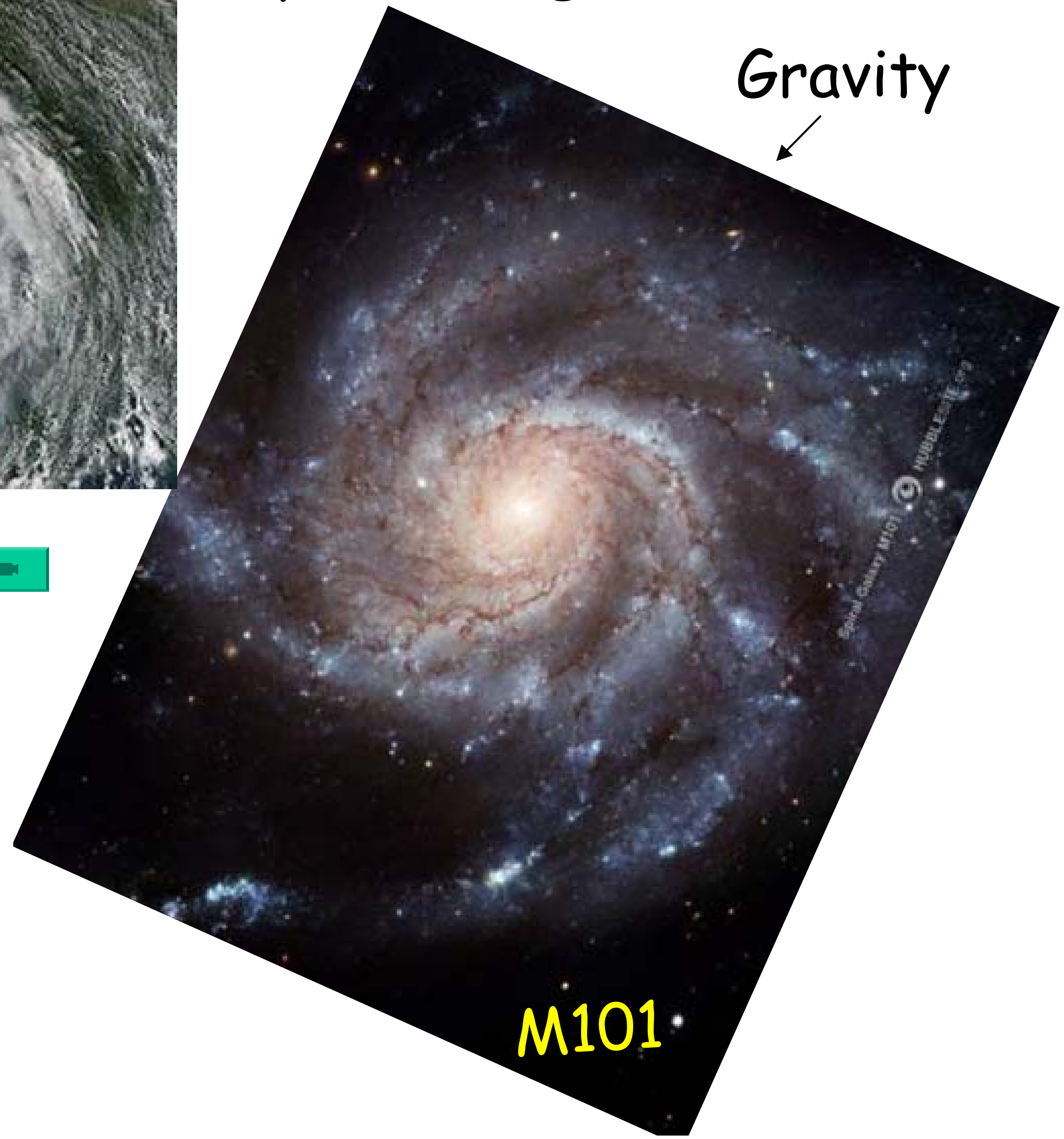
- Estimated the rotation speed of the universe by comparing with data: $\Omega \sim 10^{-13}$ rad yr^{-1}



Infall of matter:

Low pressure center,
pressure gradient

Gravity



Are these both due to
Coriolis force?

A pseudo force due to
the rotating frame

Images courtesy NASA

CMB Constraint of Rotating Universe

- Treat the rotation as perturbations; use CMB data to constrain rotation speed Ω

Bianchi models (1^{st} order effect) uniform Ω

- For closed universe (Hawking 1969)
  - $< 10^{-14}$ - 7×10^{-17} rad/yr
- For open universe (Hawking 1969)
  - $< 2 \times 10^{-17}$ rad/yr
- For flat universe (Barrow et al. 1985)
  - $< 1.5 \times 10^{-15}$ rad/yr

S. W. Hawking, MNRAS **142**, 129 (1969). J. D. Barrow *et al:*, MNRAS **213**, 917 (1985).
C. B. Collins and S. W. Hawking, MNRAS **162**, 307 (1973).

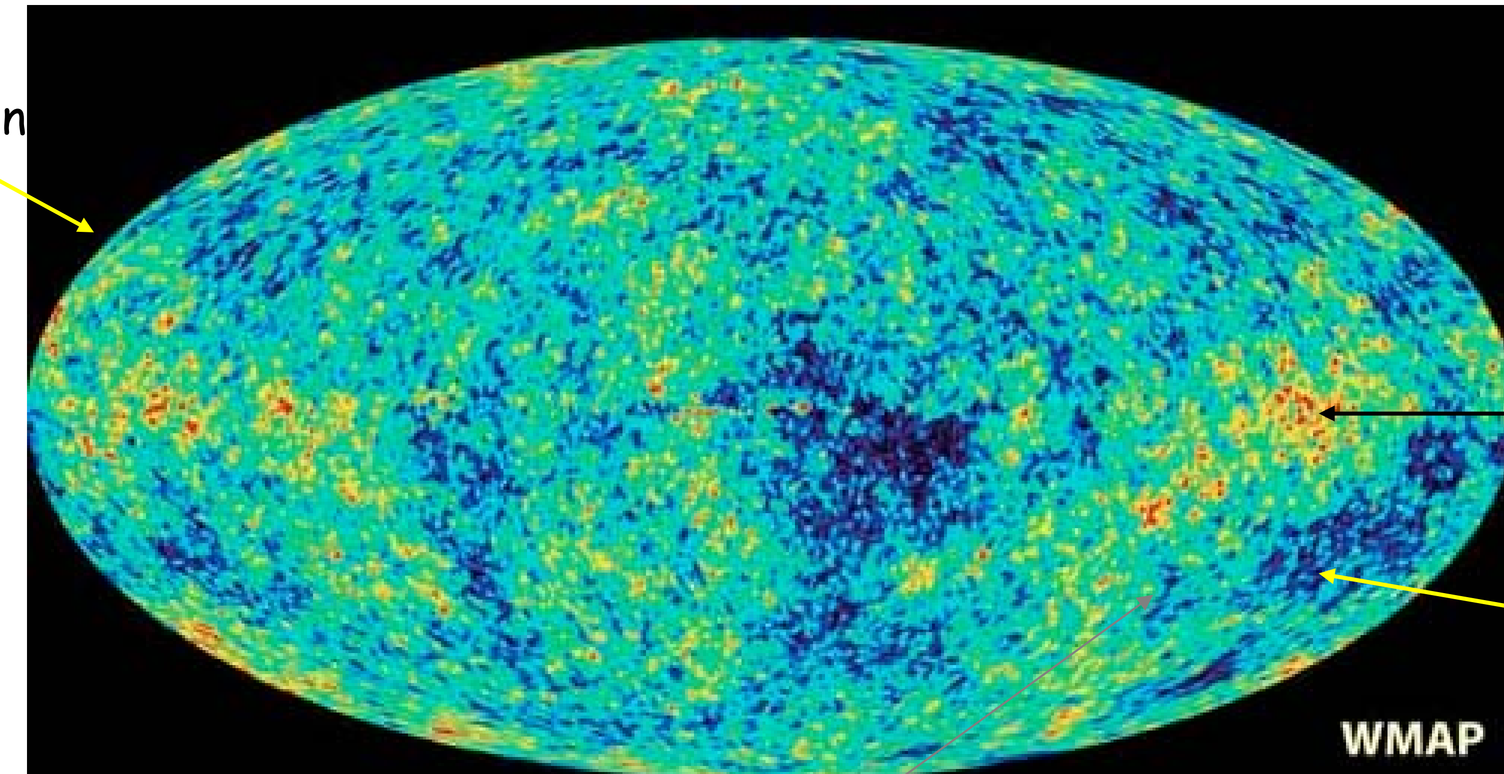
Inflation model: exponential expansion of the universe dampens out any initial rotation: Ω = 0!

J. Ellis and K. A. Olive, Nature **303**, 23 (1983).

Cosmic Microwave Background (CMB) Anisotropies

Emitted at $t \sim 400{,}000$ yrs, farthest EM signals we can observe!

All-sky projection



hotter

colder

$$T = 2.725 \pm 10^{-5}\ \mathrm{K}$$

CMB Anisotropies = Temperature fluctuations $\sim 10^{-5}\,\mathrm{K}$

Figure courtesy NASA/WMAP

Axis Of Evil

Is there a preferred direction
in the universe?

Data analysis:

- M. Tegmark *et al.*, PRD **68**, 123523 (2003).
- D. J. Schwarz *et al.*, PRL **93**, 221301 (2004).
- De Oliveira-Costa *et al.*, PRD **69**, 063516 (2004).
- K. Land and J. Magueijo, PRL **95**, 071301 (2005).
- M. J. Longo, *arXiv:astro-ph/0703325*

Theory:

- Luminet *et al., Nature* **425**, 593 (2003).
- L. Campanelli *et al.*, PRL **97**, 131302 (2006).
- M. J. Longo, *arXiv:astro-ph/0703694*
- L. Campanelli *et al., arXiv:astro-ph/0706.3802v2*

‘Axis of Evil’?

Galactic coordinates:

(l ~-90^o, b ~ 50^o)

longitude latitude

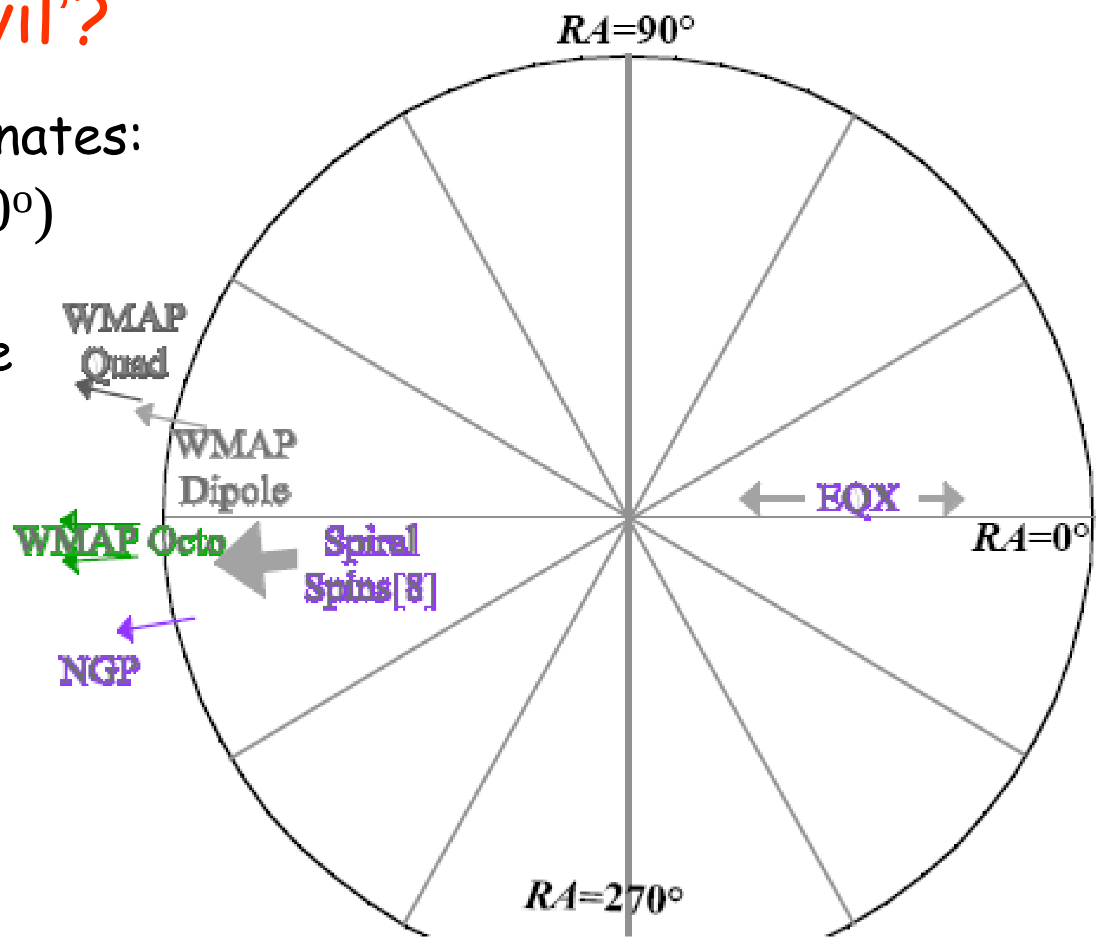
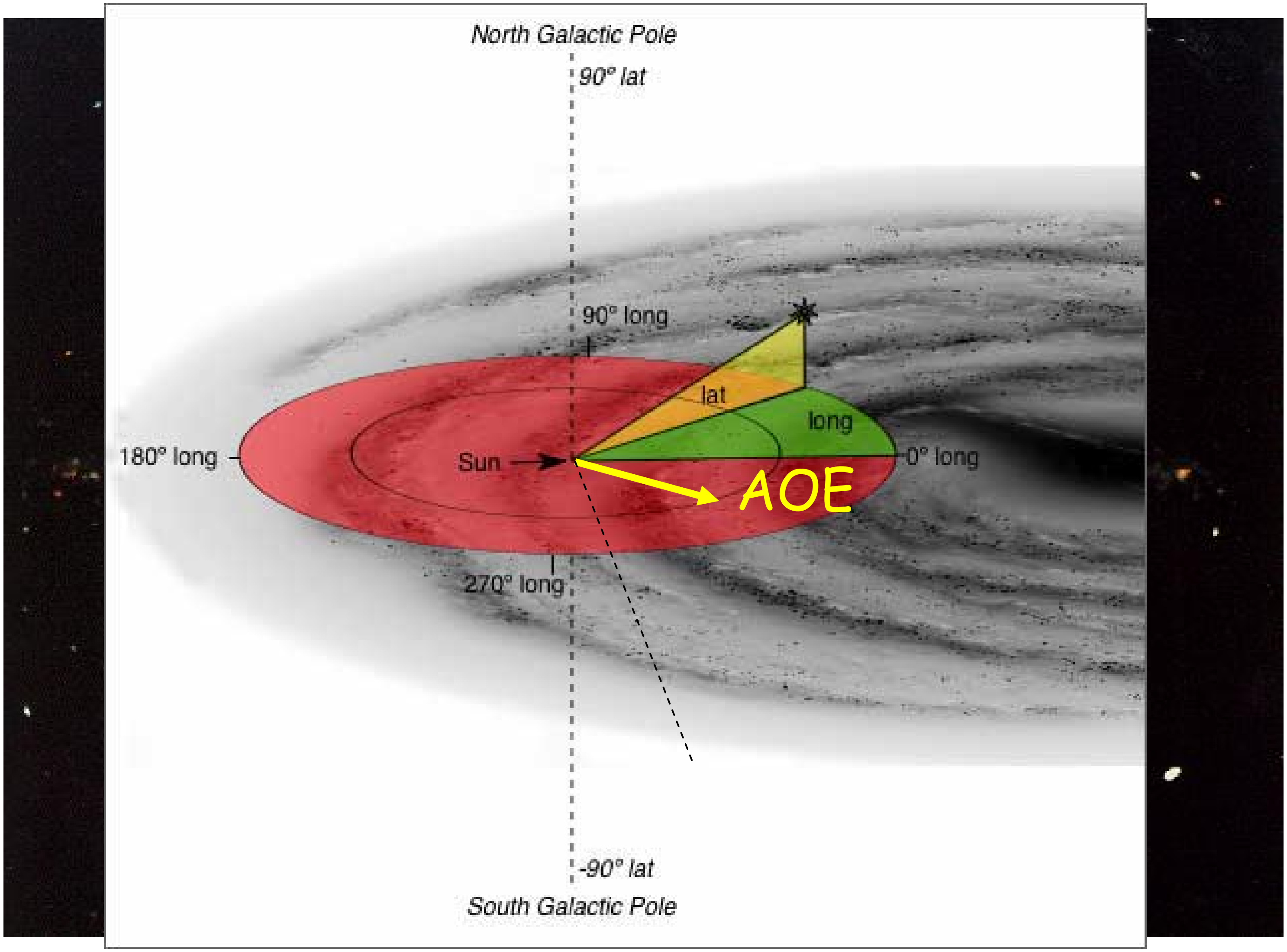


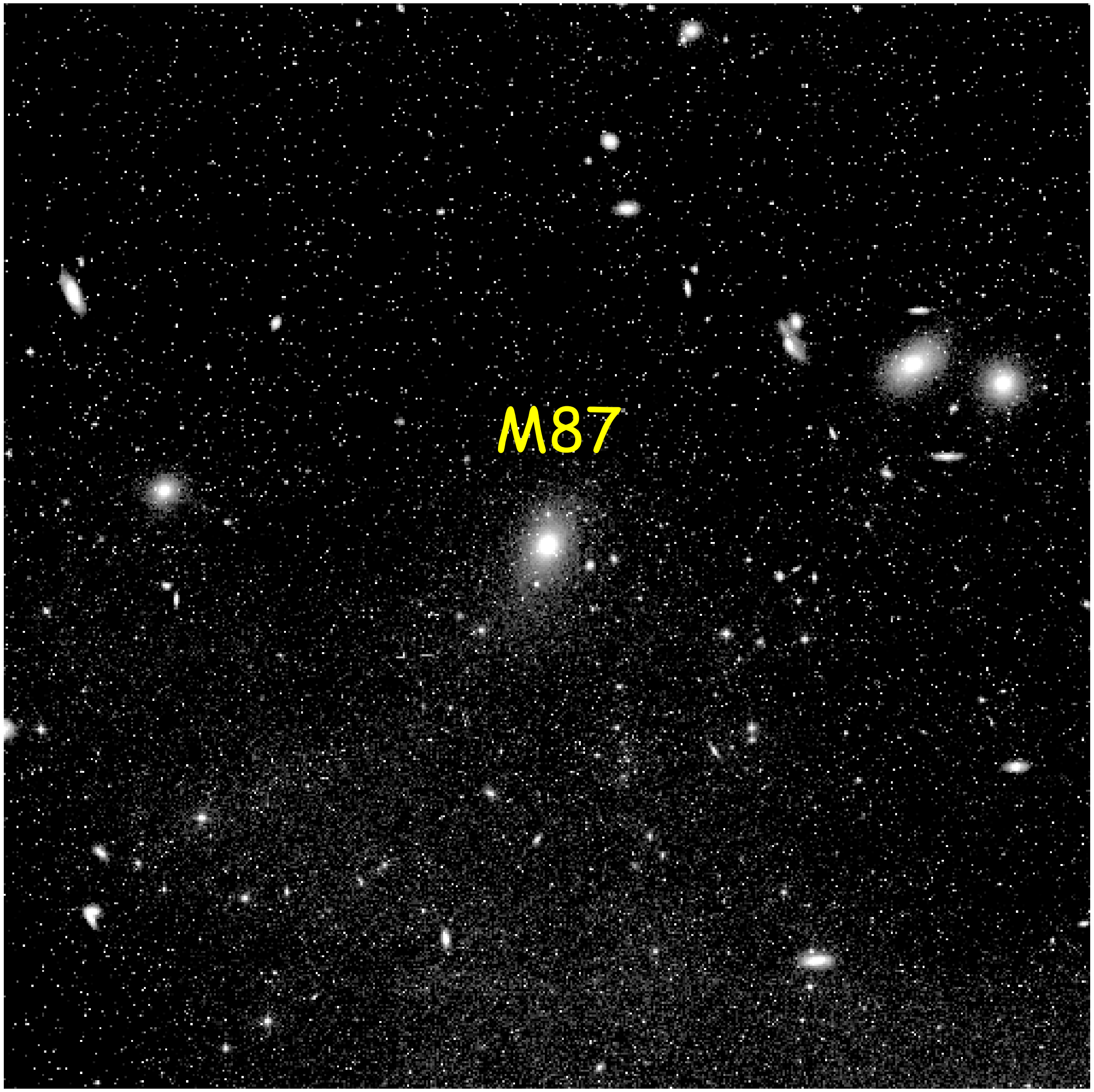
Fig. 2. Right ascensions of the alignments discussed in the text. The WMAP dipole, quadrupole, and two of the octopoles are indicated. NGP is the North Galactic pole. EQX are the equinoxes. The axis of the spiral galaxy spin asymmetry from [8] is also shown. The declinations of all these alignments are within about ±15° of each other and are ~0°.

M. J. Longo, arXiv:astro-ph/0703694

Milky Way


North Galactic Pole
90° lat
90° long
lat
long
180° long
Sun
0° long
AOE
270° long
-90° lat
South Galactic Pole

AOE points towards Virgo Cluster



distance ~ $6x10^7$ l.y.s,
> 2000 galaxies.
Milky Way is being drawn there at several hundred km/s.

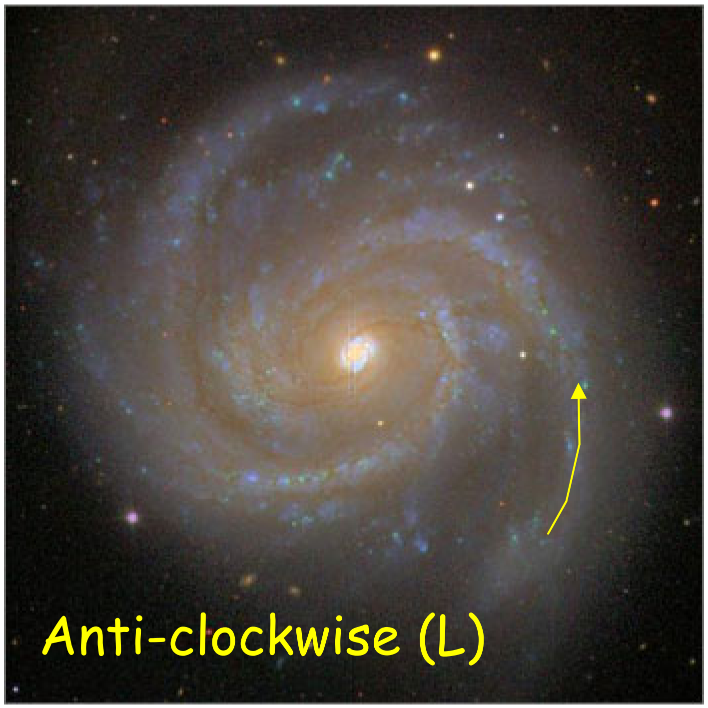
Photo credit: Digitized Sky Survey, Palomar Observatory, STScI

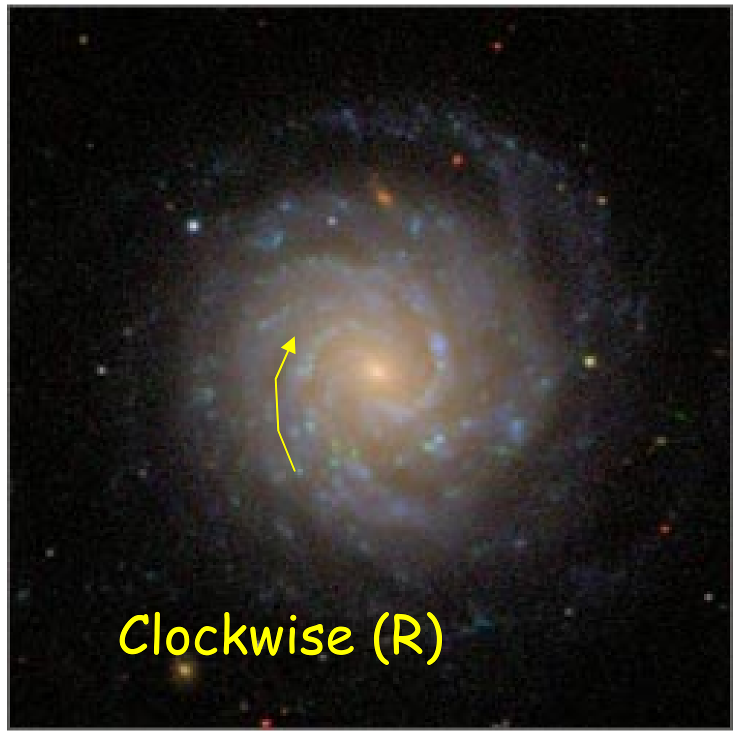
M. J. Longo, arXiv:astro-ph/0703325

Table I. Number counts and net asymmetries for the RA ranges indicated. The 3rd row gives the combined numbers for the first two rows. The last column gives the number of standard deviations for the asymmetries.

RA Range	N^+	N^-	N_{Tot}	$(N^+ - N^-)/N_{\mathrm{Tot}} \pm \sigma$	$\langle A \rangle / \sigma$
(R) 80° to –80°	118	104	222	0.063±0.067	+0.94
(L) 150° to 210°	296	368	664	–0.108±0.039	–2.79
(R-L) Combined	178	264	886	–0.0971±0.0336	$\boxed{-2.89}$

Prob. < 0.39%





Parity violation?

Project Galaxyzoo

http://www.galaxyzoo.org/Project2.aspx

Count the spirals; determine whether there's AOE!

http://news.bbc.co.uk/2/hi/science/nature/6289474.stm

Finally, our work …

Shi Chun Su and M.-C. Chu, ‘Is the Universe Rotating?’, ApJ **703**, 354 (2009).

ApJ **703**, 354 (2009).

Our Model

$$ds^2 = a^2(\eta)\{[1-\boxed{f(r,\eta)}]d\eta^2 - [1-\boxed{h(r,\eta)}]dr^2 - [1-\boxed{h(r,\eta)}]r^2 d\theta^2$$
$$-[1-\boxed{k(r,\eta)}]dz^2 + \boxed{2r^2 a(\eta)\Omega(r,\eta)d\theta d\eta}\},$$

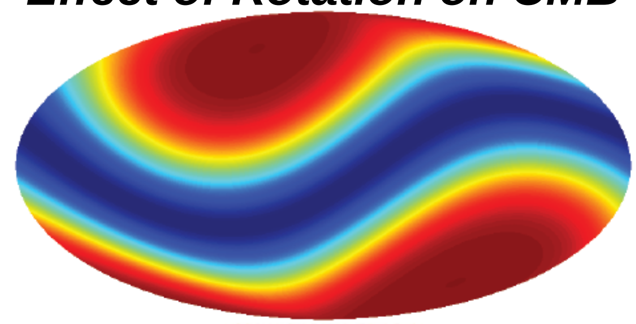
Add rotational perturbations to the flat RW model (standard model)

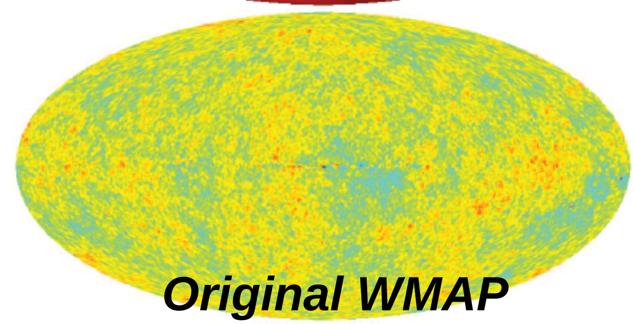
- $\Omega(r, \eta)$: angular velocity
  - Allows non-uniform rotation
- Flat RW model
  - Supported by observation
- 2nd order perturbations
  - Parity symmetry

$f(r,\eta)$, $h(r,\eta)$ and $k(r,\eta)$

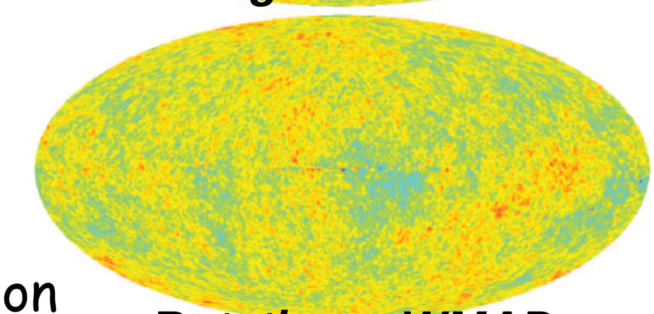
Constraint: $\boxed{\Omega < 10^{-9} \text{ rad yr}^{-1}}$ at CMB emission

Effect of Rotation on CMB





Original WMAP



Rotation + WMAP

CMB Constraint of Rotating Universe

- Treat the rotation as perturbations; use CMB data to constrain rotation speed Ω

Bianchi models (1^{st} order effect) uniform Ω

- – For closed universe (Hawking 1969)
  - < 10^{-14} - 7×10^{-17} rad/yr
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  - < 1.5×10^{-15} rad/yr

RW model (2^{nd} order) $\Omega(r)$

- For flat ΛCDM universe (Su and Chu 2009)

$\boxed{< 10^{-9} \text{ rad/yr}}$

Cf. Li: $\Omega \sim 10^{-13}$ rad/yr

Looser bound: could we revive Gamow, Li's dream? Explain AOE?

…to be continued…

Summary

I. If the universe rotates:

- May explain the rotation of most stellar objects
- But may violate causality
- May impact our understanding of time (time travel possible?)
- May need to throw away inflation theory, revise standard cosmological theory

II. Using data to constrain rotation speed: highly dependent on model, not conclusive yet

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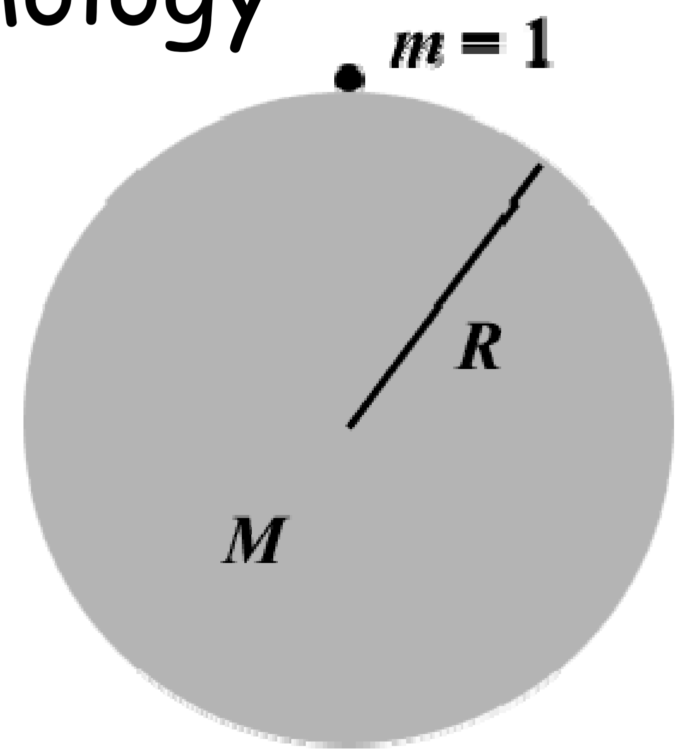
Department of Physics
The Chinese University of Hong Kong

Newtonian Cosmology



- A spherical universe of uniform density
  $\rho = 3M/4\pi R^3$
- Dynamics of a unit mass on 'surface':

$$\boxed{\frac{1}{2}\left(\frac{\dot{R}}{R}\right)^2 = \frac{4\pi G}{3}\rho + \frac{\eta}{R^2}}$$

$$\boxed{\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\rho}$$

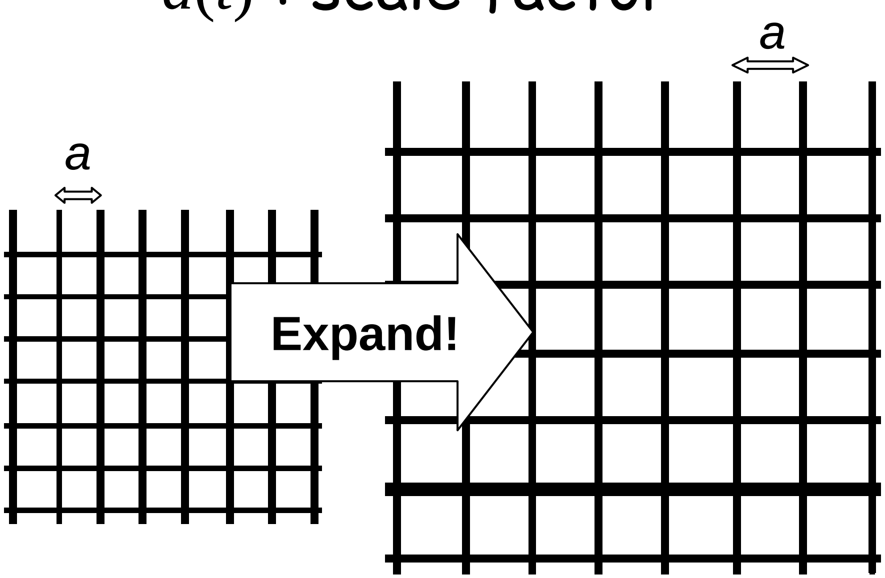
– $\eta < 0$: expansion will slow down to stop, and then the universe collapses
- $\eta = 0$: expands exactly at the "escape velocity"; expansion will slow down but *continues forever*
- $\eta > 0$: expansion continues forever

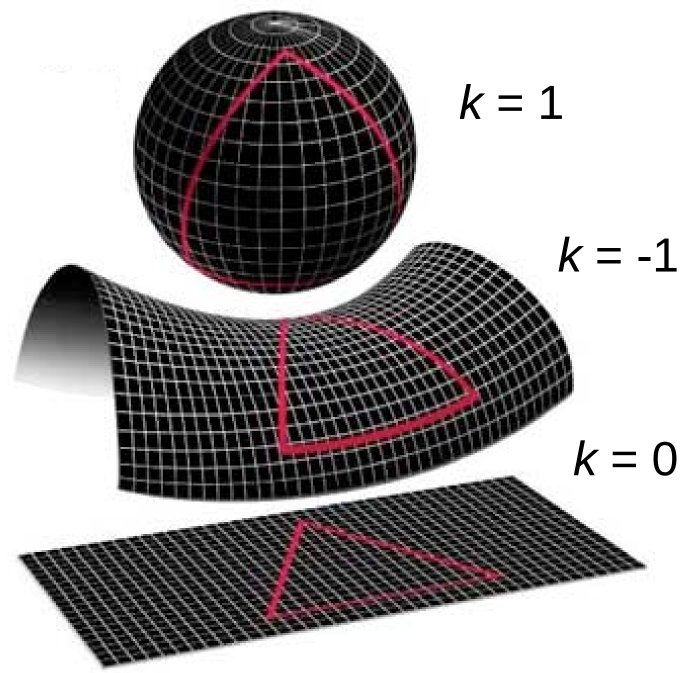
Robertson-Walker Metric

- Most general metric (non-rotating)
  - Homogeneity and isotropy

$$g_{\mu\nu}dx^\mu dx^\nu = \quad -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right]$$

  - k = -1 (open), 0 (flat), 1 (closed)
  - $a(t)$: scale factor





http://www.universeadventure.org/fundamentals/images/model-spacetimegeometry.jpg

Mach's Principle

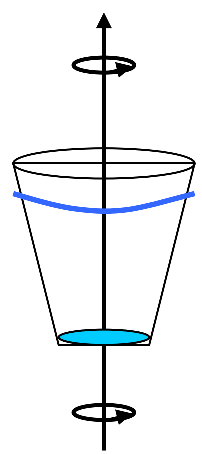
BUT: rotation with respect to what?

Rotation w.r.t. space is ok if space is absolute (Newtonian)!

Mach's Principle: motion is meaningful only relative to other bodies; ~ 'the inertia of a body is determined in relation to all other bodies in the universe'

There's nothing *outside* of the universe, no other reference!

Uniform rotation of the universe violates Mach's Principle!



Water surface curved when the bucket is rotating.

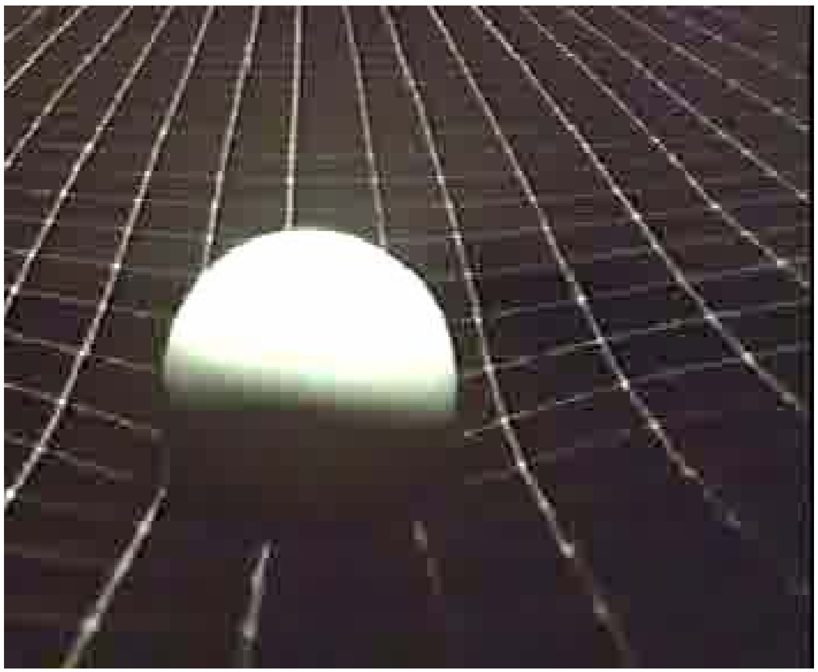
Newton: rotation is relative to absolute space.

Mach: rotation is relative to distant stars.

Rotation in General Relativity

In Newtonian Physics space is absolute, ok to talk about rotation of matter w.r.t. space

GR: space is relative, deformable by matter/energy



invariant interval:

$ds^2 = dt^2 - dx^2 - dy^2 - dz^2$

flat non-rotating space-time

$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2$

polar coordinate

$ds^2 = dt^2 - dr^2 - r^2 d\theta^2 - dz^2$

Rotating with angular speed Ω:

$ds^2 = dt^2 - dr^2 - r^2(d\theta - \Omega dt)^2 - dz^2$

$ds^2 = (1 - r^2\Omega^2)dt^2 - dr^2 - r^2 d\theta^2 - dz^2$

$+ 2r^2\Omega d\theta dt$

Distortion of space-time due to rotation

Analytic Solutions of the EFEs

- $$ {}_2u_0(r,\eta) = \frac{a(\eta)}{2}\left\{-2\,{}_1u_2\Omega(r,\eta) + \frac{[{}_1u_2(r,\eta)]^2}{r^2a^2(\eta)} + k(r,\eta) + T(r,\eta)\right\} $$

- $$ \begin{aligned} {}_2u_1(r,\eta) = {} & \frac{-a^2(\eta)}{8\dot{a}^2(\eta) - 4a(\eta)\ddot{a}(\eta)}\{-\dot{a}(\eta)[2k'(r,\eta) + 2T'(r,\eta)] \\ & +a(\eta)[-2\dot{k}'(r,\eta) + \dot{L}'(r,\eta)]\}, \end{aligned} $$

- $$ \begin{aligned} {}_2\rho(r,\eta) = {} & -\frac{1}{32\pi a^2(\eta)}\{-4\Lambda a^2(\eta)[k(r,\eta) + T(r,\eta)] \\ & + \frac{4\dot{a}(\eta)[3\dot{k}(r,\eta) - 2\dot{L}(r,\eta)]}{a(\eta)} - \frac{2[2k'(r,\eta) - L'(r,\eta)]}{r} \\ & +12ra^2(\eta)\Omega(r,\eta)\Omega'(r,\eta) + r^2a^2(\eta)\Omega'^2(r,\eta) - 4k''(r,\eta) \\ & +2L''(r,\eta) + 4r^2a^2(\eta)\Omega(r,\eta)\Omega''(r,\eta) \\ & +4\left[\Lambda^2(\eta) + \frac{3\dot{a}^2(\eta)}{a^2(\eta)}\right]\{k(r,\eta) + T(r,\eta)\} \\ & -r^2a^2(\eta)\Omega^2(r,\eta) + \left[ra(\eta)\Omega(r,\eta) - \frac{{}_1u_2(r,\eta)}{ra(\eta)}\right]^2\}\}, \end{aligned} $$

$$ \rho = \cancel{{}_0\rho} + \cancel{{}_1\rho} + \cancel{{}_2\rho}, $$

$$ P = \cancel{{}_0P} + \cancel{{}_1P} + {}_2P, $$

0,1,2,3 0,1,2,3

$$ u^\mu = \cancel{{}_0u^\mu} + \cancel{{}_1u^\mu} + \cancel{{}_2u^\mu}, $$

$\boldsymbol{T}(r,\eta)$, $\boldsymbol{L}(r,\eta)$ and $k(r,\eta)$

Analytic Solutions of the EFEs

- ${}_2P(r,\eta)$: 3 equations for each spatial direction
  - Same $\Omega(r,\eta) = a^{-3}(\eta)A(r) + B(\eta)$ $r,\eta) =$ unknown k, T, L
  - W

$$T'(r,\eta) - rT''(r,\eta) = -\frac{2r^3A'^2(r)}{a^4(\eta)} - \frac{\Omega_\Lambda r[3A'(r)+rA''(r)]^2}{2\Lambda(1-\Omega_\Lambda)a^3(\eta)},$$

$$\frac{r}{a(\eta)}\frac{d}{d\eta}[a^2(\eta)\dot{L}(r,\eta)] - a(\eta)[L'(r,\eta)+rL''(r,\eta)]$$
$$= \quad ra(\eta)T''(r,\eta) - r^3a^{-3}(\eta)A'^2(r).$$

  - k is absent, freedom! Set $k = 0$
  - homogeneous rotation B is absen

$$\begin{aligned} \rho &= \cancel{{}_0\rho} + \cancel{{}_1\rho} + \cancel{{}_2\rho}, \\ P &= \cancel{{}_0P} + \cancel{{}_1P} + \boxed{{}_2P}, \\ u^\mu &= \cancel{{}_0u^\mu} + \overset{0,1,2,3}{\cancel{{}_1u^\mu}} + \overset{0,1,2,3}{\cancel{{}_2u^\mu}}, \end{aligned}$$

$$\boxed{T(r,\eta),\ L(r,\eta)} \text{ and } \cancel{k(r,\eta)}$$

Analytic Solutions of the EFEs

- Taylor series + separation of variables
  - Let $A(r) = \Omega_M \sum_{n=2}^{\infty} c_n r^n \qquad L(r,\eta) = \Omega_M^2 \sum_{n=0}^{\infty} E_n(\eta) r^n$

$$
\begin{aligned}
T(r,\eta) &= -\frac{\alpha^2\Omega_M^2}{3a^4(\eta)} r^6 - \frac{4\Omega_\Lambda \alpha^2 \Omega_M^2}{\Lambda(1-\Omega_\Lambda)a^3(\eta)} r^4, \\
0 &= E_{2n+1}(\eta), \\
0 &= \frac{1}{a(\eta)}\frac{d}{d\eta}[a^2(\eta)\dot{E}_0(\eta)] - 4a(\eta)E_2(\eta), \\
-\frac{48\Omega_\Lambda \alpha^2}{\Lambda(1-\Omega_\Lambda)a^2(\eta)} &= \frac{1}{a(\eta)}\frac{d}{d\eta}[a^2(\eta)\dot{E}_2(\eta)] - 16a(\eta)E_4(\eta), \\
-\frac{14\alpha^2}{a^3(\eta)} &= \frac{1}{a(\eta)}\frac{d}{d\eta}[a^2(\eta)\dot{E}_4(\eta)] - 36a(\eta)E_6(\eta), \\
&\vdots \\
0 &= \frac{1}{a(\eta)}\frac{d}{d\eta}[a^2(\eta)\dot{E}_{2n-2}(\eta)] - 4n^2 a(\eta)E_{2n}(\eta).
\end{aligned}
$$

Recursion relation!

$$
\begin{aligned}
\rho &= \cancel{{}_0\rho} + \cancel{{}_1\rho} + \cancel{{}_2\rho}, \\
P &= \cancel{{}_0P} + \cancel{{}_1P} + \cancel{{}_2P}, \\
u^\mu &= \cancel{{}_0u^\mu} + \cancelto{0,1,2,3}{{}_1u^\mu} + \cancelto{0,1,2,3}{{}_2u^\mu},
\end{aligned}
$$

$\cancel{T(r,\eta)}$, $\cancel{L(r,\eta)}$ and $\cancel{k(r,\eta)}$

Its 2nd-Order Sachs-Wolfe Effects

- 1st order:

$$\begin{aligned}\frac{\delta T_O}{T_O} &= -\frac{1}{2}{}_1g_{00}|_O^\epsilon + {}_0k^2({}_1g_{02} + u_2)|_O^\epsilon - {}_1k^0|_O^\epsilon \\ &= 0 \qquad \textit{Expected}!\end{aligned}$$

- 2nd order:

$$\begin{aligned}\frac{\delta T_O}{T_O} &= \left[-\frac{{}_2u_1(r_\lambda,\eta_\lambda)}{a(\eta_\lambda)}\sin\phi + \frac{{}_2u_0(r_\lambda,\eta_\lambda)}{a(\eta_\lambda)} +_2 k^0(\lambda) + \frac{{}_1u_2(r_\lambda,\eta_\lambda)}{a(\eta_\lambda)}{}_1k^2(\lambda)\right]\Big|_{\eta_\epsilon}^{\eta_0} \\ &= \int_{\eta_\epsilon}^0 \left[-\frac{\dot{T}(-\lambda\sin\phi,\lambda)}{2} + T'(-\lambda\sin\phi,\lambda)\sin\phi - \frac{\dot{L}(-\lambda\sin\phi,\lambda)}{2}\sin^2\phi\right]d\lambda \\ &\quad -\frac{1}{2}\left\{\frac{\Omega_\Lambda^2 a^4(\eta_\epsilon)}{4\Lambda^2(1-\Omega_\Lambda)^2}[3\Omega'(r_\epsilon,\eta_\epsilon) + r_\epsilon\Omega''(r_\epsilon,\eta_\epsilon)]^2 + T(r_\epsilon,\eta_\epsilon)\right\} \\ &\quad +\frac{\Omega_\Lambda\sin\phi}{2\Lambda(1-\Omega_\Lambda)}[2\dot{a}(\eta_\epsilon)T'(r_\epsilon,\eta_\epsilon) - a(\eta_\epsilon)\dot{L}'(r_\epsilon,\eta_\epsilon)],\end{aligned}$$

What we need!!

Constraints on Our Model

- Simplest case (homogeneous rotation):

$$\Omega(r,\eta) = B(\eta)$$

  – 2nd order SW eff $\frac{\delta T_O}{T_O} = 0$

$$_1u^2(r,\eta) = \Omega(r,\eta)$$

  – Only the difference is important!

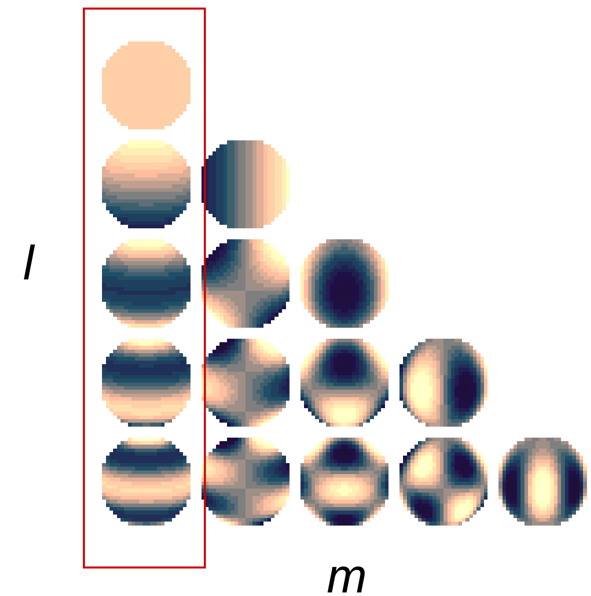
Effect of the rotating metric ⟺ **Cancel** ⟺ **Relativistic Doppler effect** caused by the sources rotating in a stationary metric

Constraints on Our Model

- Second Trial: $\Omega(r,\eta) = \Omega_M a^3(\eta_\epsilon)\boxed{r^2}/(r_\epsilon^2 a^3(\eta))$

$$\begin{aligned}\frac{\delta T_O}{T_O} &= a_2 \sin^2\phi + a_4 \sin^4\phi + a_6 \sin^6\phi \\ &= \boxed{A_0} Y_0^0(\phi,\theta) + \boxed{A_2} Y_2^0(\phi,\theta) + \boxed{A_4} Y_4^0(\phi,\theta) + \boxed{A_6} Y_6^0(\phi,\theta).\end{aligned}$$

- Cannot explain alignments of multipoles in our example
- With $\Omega_\Lambda = 0.742,\ H_0 = 71.9\mathrm{km/s/Mpc}$ and $A_n \sim 10^{-5}$
- Constraint $\sim 10^{-9}$ rad yr^{-1}
  - At the last scattering surface
  - When photons decoupled



http://en.wikipedia.org/wiki/Spherical_harmonics

Examination of Evidence for a Preferred Axis in the Cosmic Radiation Anisotropy

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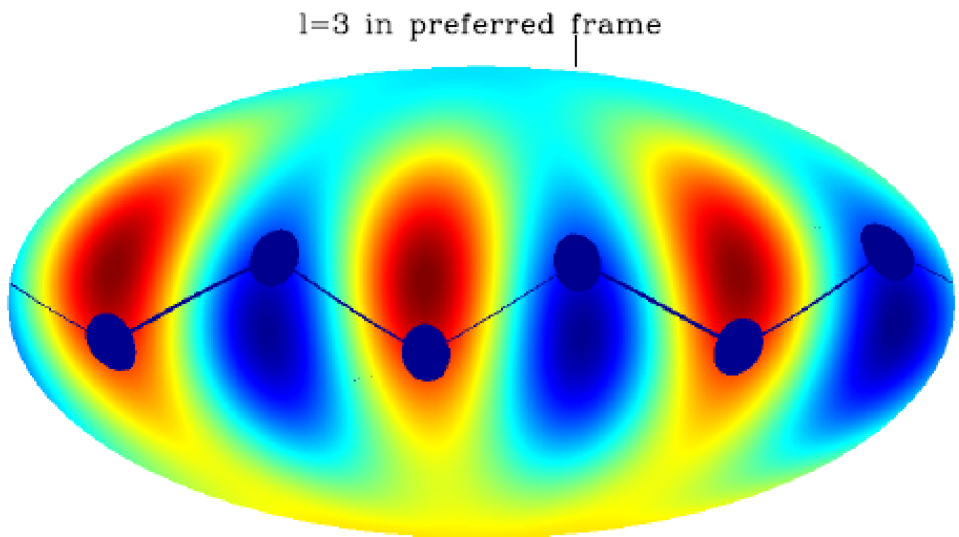
We examine previous claims for a preferred axis at $(b, l) \approx (60, -100)$ in the cosmic radiation anisotropy, by generalizing the concept of multipole planarity to any shape preference. Contrary to earlier claims, we find that the amount of power concentrated in planar modes for $l = 2, 3$ is not inconsistent with isotropy and Gaussianity. The multipoles' alignment, however, is indeed anomalous, and extends up to $l = 5$ rejecting statistical isotropy with a probability in excess of 99.9%. There is also an uncanny correlation of azimuthal phases between $l = 3$ and $l = 5$. We are unable to blame these effects on foreground contamination or large-scale systematic errors. This reappraisal may be crucial in identifying the theoretical model behind the anomaly.



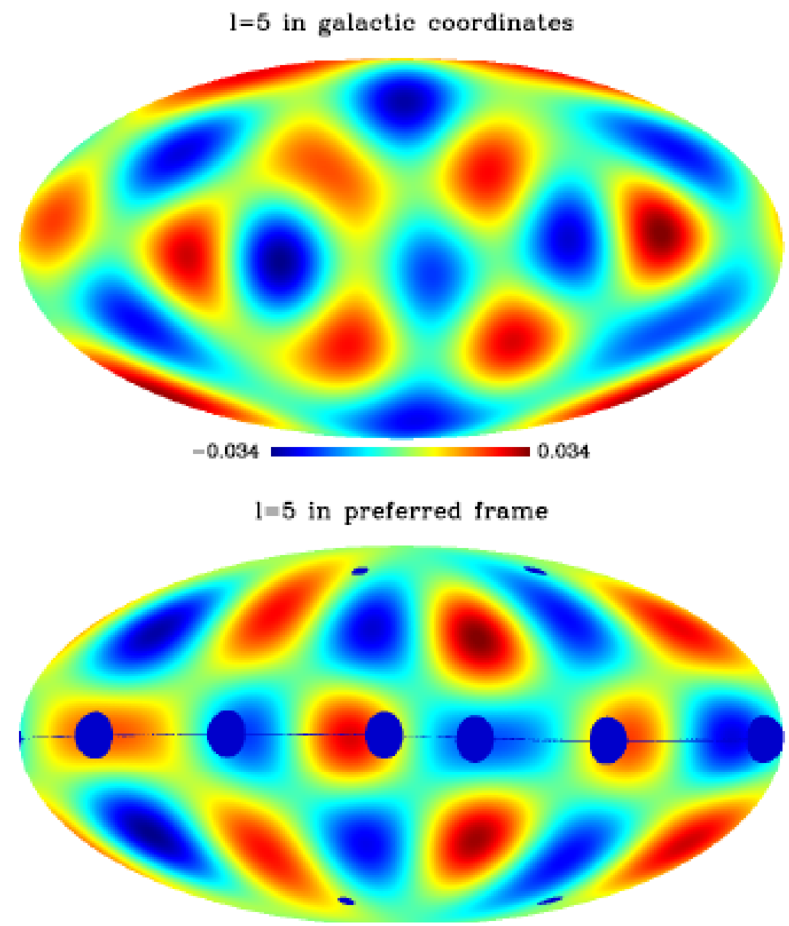


FIG. 1 (color online). The $l = 5$ multipole in galactic coordinates (top) and aligned with $(b, l) \approx (50, -91)$ (middle), and the $l = 3$ multipole in its preferred frame. We have superposed the multipole vectors, and the chain linking them.