Why Use Mathematization? And Why Not?— Superiorities and Limitations of the Mathematization in Science

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I. Introduction

No other episode in the history of science has been as crucial as the beginning of mathematization, both owing to the significance of its impact on the development of science during the scientific revolution over the past few centuries, but also with reference to the debates about its superiorities and limitations generated among scholars, those who have labored to understand the nature (Gorham *et al.* 1). Descartes, a French mathematician and pioneer of science, proclaimed explicitly that the essence of nature was mathematics and all principles of nature could be represented in mathematics (Kline, 1: 326), while another noted mathematician and physicist Weyl defined mathematization as a "creative activity of man, . . . , whose historical decisions defy complete objective rationalization." (3: 1210)

In this paper, the characteristics of mathematization would be explained as well as its corresponding superiorities and limitations.

On top of the superiorities and limitations, some viewpoints about the future mathematization of science would also be stated.

(I) The Definition and Characteristics of Mathematization

Though the idea of mathematization has ancient roots, the word "mathematization" was firstly used in 1908 to provide a description for the tendency to involve mathematical thoughts. ("Mathematization") Central as the process of the application of concepts, procedures and methods developed in mathematics to other fields of science, mathematization has promoted the exploration of nature and the development of science with three characteristics: quantification, axiomatization and idealization, which would be discussed in detail in the following article (Roux 324).

II. Superiorities of Mathematization

(I) Superiorities of Quantification

Quantification is the act of counting and measuring that maps human observations and experience into quantities. These numbers would represent the quantity of the property associated with the concept, which would favor further mathematical process such as calculations or comparison (Ma 1). Accurate predictions of future situations, enabled by quantification, would thus become falsifiable.

Take "length" as an example. If it is going to be checked that whether rope A is two times longer than rope B, a ruler should be confirmed to provide a linear scale, namely a unit, to quantify the concept "length". Thus, "length" would become objective and concrete, as the result of which the equation $a=2b$ could be proved convincingly[1]. From this

1 a, b mean the length of rope A and rope B

example, it also could be found that quantification is the basis for further functional process.

Precise predictions of future situations with existing data would be enabled by quantification, which could be seen in the example of so-called Halley's comet. This famous prediction on the period of a comet was made on the calculus and applications of previous observation results and then amazed the people at that time. It could be easily proved by observation, although at that time Halley had already died. This successful prediction symbolized the beginning of the age of faith in science with the idea of quantification, without which reliable predictions would not have been made (Cohen 62).

(II) Superiorities of Axiomatization

Axiomatization means to find the parallel structure between the axiomatic system and a science. And in mathematics, an axiomatic system is a sequence of axioms from which some or all axioms can be used in conjunction to logically induce corresponding theorems. It could reveal precisely what assumptions underlie which branch, and a comparison and clarification of the relationships of various branches could be made, which would be fruitful in many domains such as physics or biology.

Euclid's works were extraordinary examples of this system, where there are definitions, postulates and common notions as premises, which could be used as bricks to build an unbreakable sky-scraper of other theories and propositions with logic as concrete (Euclid 275–277). In Newton's book, there is an imitation, or would rather say, a salute to Euclid's *Elements*, showing that Newton advocated the axiomatization of physics. Newton was using definitions to derive the consequences of axioms, which he called the laws of motion (67). And he succeeded, giving a simple but also exquisite description of the nature.

Moreover, Newton's works were not the end in the axiomatization of physics. His successors, D'Alembert and Lagrange, who are also extraordinary mathematicians, proceed with his research and find an equation called "Least Action Principle", which could be roughly seen as the generalization origin of all physics theorems with its extensive utilization in nearly all branches of science. ("D' Alembert-Lagrange Principle")

If there were only oceans of independent complicated laws, it would be time-consuming and laborious to understand the nature. On the basis of axiomatic system, the unity of those indispensable, closely-related and interweaved knowledge would become more easily accessible.

(III) Superiorities of Idealization

The processes of idealization have two parts: one ascending from the life-world, the other descending and applying to it. The former ascending movement would be referred to as idealization_1, and the latter descending movement will be referred to as idealization_2 (Garrison 330). In general, the purpose of idealization_1 is to abstract the complex world to derive the ideal models and laws from the life-world, and the idealization_2 is to apply those ideal models or laws to the life-world.

According to Poincaré, simple facts always lie in the infinitely great and in the infinitely small (163). However, those two ideal cases could not be found in real-world. Thus, we need to idealize_1 a world where there are "infinitely great" and "infinitely small" to assist us unveil and understand the science behind the facts, and then apply the rules we established to discover similarities under apparent discrepancies. Here, the world opposite to the life-world would also strictly follow the law of mathematics. It's not a coincidence, but a fact owing to the aforementioned superiorities of quantification and axiomatization. Hence, we could make accurate predictions and deductions.

One famous example is the Galilean analysis of freefall. Galileo's purpose was to disprove the widely accepted common sense view of the Aristotelians that the velocity of a freefall object was determined by the mass of that. After the observations and a series of tightly interwoven experiments, Galileo found that the speed between two things differ less and less despite extreme difference in weight. Hence, the ideal case he needed was "infinitely small resistance", the so-called vacuum. Obviously, the ideal case could not be found in nature, but he could idealize a prefigured world of experiments to substitute for it.

III. Limitations of Mathematization

(I) Too Far Away—The Loss of Authenticity

In an axiomatization system, it is established that if the axioms were correct, then their deductions, constructed through rigid logic, would be correct as well. In this way, many concepts, created without regard to transient reality, could be introduced and proved but may nevertheless be useful, as in the case of 4-dimensional geometry[2].The introduction and acceptance of those concepts, which have no immediate counterparts in the real world, gradually convinced us that mathematics is a human creation, rather than a mapping space of the reality in nature, which is derived solely from nature (Kline, 3: 1024). Since the concepts are severed from the

2 Questions and researches on 4-dimensional world are relatively important in physics, especially in the field of General Relativity. But the fact is that we human live in 3-dimensional world and may never be honored to try 4-dimensional world.

physical world and lose their claim to the truth about nature, the process of mathematization does not simplify but complicate our exploration of nature.

(II) An Overall Collapse—Paradox in Mathematic Logic

It was Euclid's neglect of postulate to justify transferring lengths that made his compass "collapsible" (Dunham 262–264). Mathematicians also worry about the collapse of the whole mathematic system, which would be catastrophe, because all our process of mathematization was involved with math. For example, if one day 1+1=2 is proven to be wrong, then all the activities, where additions will be used such as business transactions, will be stopped until it is treated stringently and redefined. This uneasiness reintroduced the process of constructive proofs (Kline, vol 3: 947). Consequently, questions and paradoxes about the validity of deductive methods were found.

One famous paradox was put as a popular form by Bertrand Russel in 1918, called the "barber" paradox[3]. A barber, who does not shave people who shave themselves, would remain in a logical predicament that he should but also should not shave himself. Both results could be induced and underpinned with his claim, though such two results could not co-exist, as long as the axiom of choice exists[4]. In other words, one object is defined in terms of a class of objects that contains the objects defined (Kline, 3: 1183). The only solution is to avoid such definitions, as Zermelo noted in 1908[5].

3 Here I will not give the explicit origin definition of that paradox in mathematics terms, as it is ambiguous and not all readers are mathematicians who could really grasp its meaning and importance.

4 Axiom of choice: For any set X of nonempty sets, there exists a choice function f defined on X. It could be understood as one input gives out exactly one output.

5 Ernst Friedrich Ferdinand Zermelo,1871–1953, was a German logician and mathematization, whose work has major implications for the foundations of mathematics such as ZFC axiomatic set theory.

This imperfect answer could not satisfy everyone, which would result in more debates in the future.

IV. Conclusion and Outlook

Mathematization has demonstrated its effect and versatility in the modern science as a constitutive element providing mathematically accurate and rigorous explanations of natural phenomena. Its characteristics as quantification, axiomatization and idealization could be found useful in the human exploration of nature. Therefore, it is not exaggerated that mathematization offers a framework following which immature sciences could be transformed into properly scientific ones.

In the further mathematization of science, attention should be paid to realizing the practical significance of science finding. And returning to the ideal of precision and rigorous proof would also be of great importance to avoid futile efforts.

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Teacher's comment:

In this essay, Qifan gave a systematic and detailed analysis on the impacts and effects of mathematization on the development of science. He attempted to argue the superiorities and limitations of mathematization based on a rich amount of scientific facts, and clarify the relationship between mathematization and science. Also, the essay is well structured and organized. Qifan showed a good understanding of the texts, which laid a solid foundation for making his arguments on this issue. (Yang Jie)