Financial Systemic Risk Prediction with Non-Gaussian Orthogonal-GARCH Models

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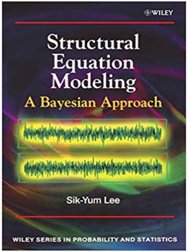
In honor of Sik-Yum Lee

whose work is well known in Structural Equation Modeling, but he has also worked on factor models in finance.

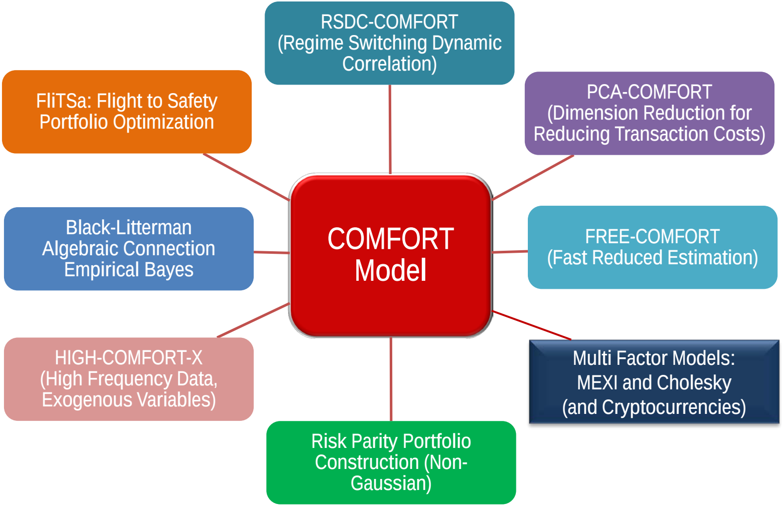


Bayesian Analysis of the Factor Model with Finance Applications, Quantitative Finance, 2007, with Wai-Yin Poon and Xin-Yuan Song

My talk is also about a factor model for financial asset returns.

Quick Tour of COMFORT: Research

COMFORT: COmmon Market Factor nOn–Gaussian ReTurn Model



Quick Tour of COMFORT: What it Does, How it Works

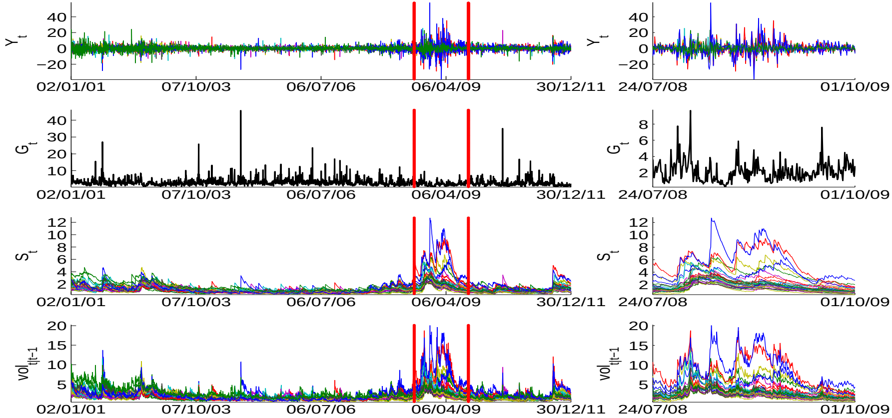
We maintain the benefits of the IID Multivariate Normal framework (estimation speed, tractability of the portfolio distribution), but deliver a superior, tractable (multivariate density) forecast.

This requires:

1. using a non-Gaussian distribution with tractable portfolio distribution,
2. new parallel, multi-step, and iterative ML estimation (EM algorithm) for the possibly hundreds of assets,
3. dynamic dependency structure (to capture "usual" volatility periods and periods with relatively high volatility and the associated high contagion),
4. addressing the multivariate GARCH problem (parametrization too large to estimate for more than a handful of assets), and
5. introduction of a Common Market Factor (a univariate latent random variable common to all stocks). This is motivated next.

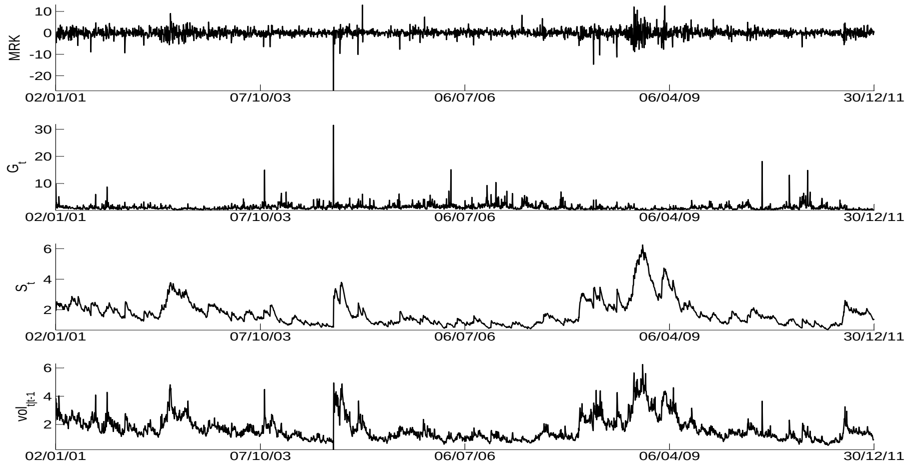
Quick Tour of COMFORT: Common Market Factor G

Returns across assets clearly have commonality in their volatility. This can be judiciously exploited. Here, the 30 DJIA stock returns, overlaid.

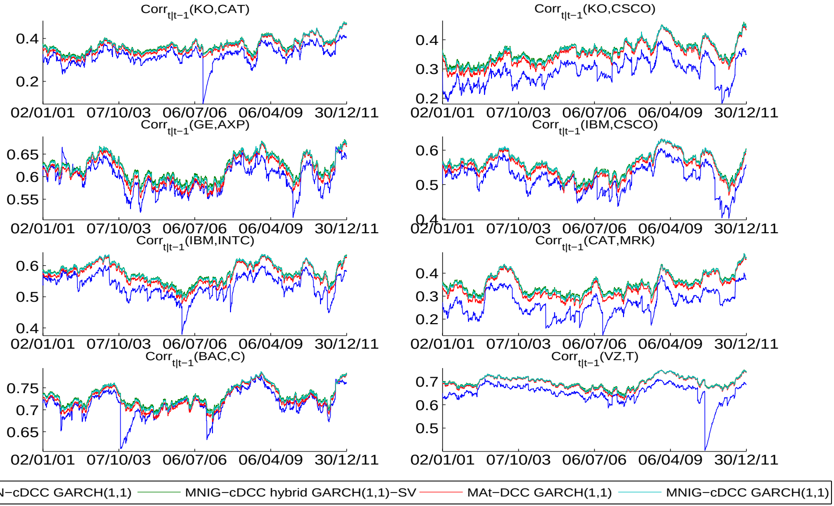


Quick Tour of COMFORT: Common Market Factor G

There are now two factors driving volatility: Persistence from GARCH, and the latent univariate "news" or "shock" process. Here, Merck:



Quick Tour of COMFORT: Comparison of Correlation Dynamics



KO: Coca-Cola; **CSCO**: Cisco; **CAT**: Caterpillar; **IBM**: International Business Machines; **INTC**: Intel; **AXP**: American Express;
GE: General Electric; **BAC**: Bank of America; **MRK**: Merck & Co; **C**: Citygroup; **VZ**: Verizon Communications; **T**: AT&T

Extension to COMFORT-PCA

Interest in Financial Systemic Risk Measures

Systemic risk refers to risk of *several* market participants suffering *severe losses simultaneously* [Benoit et al., 2017].
Following the **global financial crisis** 2008-2009 interest in measures of systemic financial risk grew considerably.

⇒ Policy makers, regulators, risk managers and investors want to detect critical risk concentrations in financial markets and to anticipate potential crises.

Plethora of measures for systemic risk in literature: According to recent survey articles several hundred measures and indicators have been proposed. Highly active research field in academia, regulatory and financial institutions. Some examples:

- *Contagion models:* [Ait-Sahalia et al., 2015], [Ait-Sahalia and Hurd, 2016], [Dungey et al., 2018], [Nasini et al., 2019]
- *Network models:* [Acemoglu et al., 2015], [Battiston et al., 2012b], [Battiston et al., 2012a], [Cont et al., 2010], [Diebold and Yılmaz, 2009, Diebold and Yılmaz, 2014], [Haldane and May, 2011],[Lenzu and Tedeschi, 2012], [Nier et al., 2007]

Motivation (2/6)

- *Correlation risk:* [Chiang et al., 2007], [Farhi and Tirole, 2012], [Lehar, 2005]
- *Tail-risk-based measures:* [Acharya et al., 2010], [Brownlees and Engle, 2016], [Brunnermeier and Cheridito, 2019], [Hautsch et al., 2014],
- *Liquidity risk:* [Brunnermeier et al., 2013], [Cifuentes et al., 2005]

Survey articles: [Benoit et al., 2017], [Bisias et al., 2012], [Brunnermeier and Oehmke, 2013], [De Bandt and Hartmann, 2000], [Fouque and Langsam, 2013], [Hansen, 2013], [Silva et al., 2017]

Problematic: Many measures are not easy to replicate and apply due to ...

- data not widely available (e.g., OTC transactions, interbank market, options)
- data not timely available (e.g., macro and fundamental data)
- require advanced statistical models (e.g., conditional tail risk measures, regime switching or BCP models).

Simpler Approach: Statistical systemic risk measure based on widely available market data (e.g., daily stock or index returns)

$\Rightarrow$ **Absorption Ratio**: Prominent risk indicator based on PCA decomposition of covariance matrix of set of asset returns; introduced in [Kritzman et al., 2011] and [Billio et al., 2012]; further investigated in [Giglio et al., 2016], [Zheng et al., 2012] and [Ren and Zhou, 2014].

Absorption Ratio (AR) measures the interconnectedness of assets by considering the fraction of total variance explained by a fixed (small) number of eigenvectors:

$$\mathrm{AR}_t = \frac{\sum_{k=1}^{\ell} \xi_{k,t}}{\sum_{k=1}^{K} \xi_{k,t}}, \qquad \xi_{1,t} \geq \cdots \geq \xi_{K,t}$$

are the ordered eigenvalues, $1 \leq \ell < K$ for K the number of assets, $\ell \in \mathbb{N}$ fixed.

(In our model, in the example below, we use $K = 30$ stocks of the DJIA, and take $\ell = 3$.)

$\Rightarrow$ AR measures risk concentration in markets; assets being tightly coupled seen as sign of fragility of the market; volatility shocks propagate faster when few factors (leading eigenvalues) explain the majority of market volatility.

Motivation (4/6)

Computation of AR in literature (and practice) uses *sample covariance estimator* on rolling windows and assumes constant covariances (and AR) per window, i.e.,

- covariance matrix, PCA decomposition and AR sensitive to outliers,
- no actual out-of-sample forecast of AR (constant),
- dynamics and reactivity of AR determined by sample size.

[Kritzman et al., 2011] suggests EWMA covariance estimator: improves dynamic evolution of AR but depends on half-life parameter (less severe).

Multivariate GARCH: parsimonious MGARCH models such as the CCC model [Bollerslev, 1990], the DCC model [Engle, 2002], VC model [Tse and Tsui, 2002] or RSDC model [Pelletier, 2006] proven successful to *model and forecast conditional covariance matrices*.

Application of (M)GARCH to risk forecasting (VaR, ES) shows (co)variance dynamics are essential for precise risk prediction and MGARCH model well suitable for this; see [Bauwens et al., 2006], [Berkowitz and O'Brien, 2002], [Kuester et al., 2006], [McAleer and Da Veiga, 2008], [Santos et al., 2013] among others.

Motivation (5/6)

More recently: MGARCH models applied to OOS portfolio optimization, e.g., [Engle and Colacito, 2006], [Luo, 2016], [Paolella and Polak, 2015b], [Santos and Moura, 2014]. Improved covariance forecasts improve performance (OOS variance, returns, Sharpe ratio) compared to IID models (at cost of higher turnover).

$\Rightarrow$ *Application of MGARCH to AR modeling and forecasting could be worthwhile...*

However...

1. Most MGARCH models are based on a Gaussian distribution (thin tails, equal tails, elliptical).
2. The CCC model takes conditional *correlations* as constant.
3. The popular DCC model (with two additional parameters no matter the dimension K) is too inflexible for large systems. The general version is infeasible in high dimensions (and hard to say if predictions are better).
4. A (non-Gaussian) MGARCH model that addresses eigenvalue dynamics directly seems natural for AR modeling.

Motivation (6/6)

O-GARCH Model: Orthogonal GARCH model from [Alexander and Chibumba, 1996, Alexander, 2001, Alexander, 2002], [Ding, 1994] assumes univariate GARCH dynamics for a few leading eigenvalues; while the remaining eigenvalues are set to zero for dimension reduction. PCA extracts the most important components of covariance dynamics. Large time-varying covariance matrix is approximated by a small number of GARCH processes.

COMFORT Model: [Paolella and Polak, 2015a, Paolella and Polak, 2016] model conditional distribution $\mathbf{Y}_t|\Phi_{t-1}$ by **MGHyp distribution** (semi-heavy-tailed, non-elliptical, mean-variance mixture) and employ CCC covariance matrix with univariate GARCH(1,1) scales; superior out-of-sample density and risk prediction.

$$\Downarrow$$

Goal: Build OGARCH model capturing all *stylized facts* of asset returns: excess kurtosis, skewness (non-ellipticity), volatility clustering, joint extremes, time varying correlations; model amenable for estimation via maximum-likelihood.

Definition: The COMFORT-PCA Model [Paolella et al., 2019]

Let the conditional return distribution admit the mean-variance mixture representation

$$\mathbf{Y}_t|\mathbf{\Phi}_{t-1} \stackrel{d}{=} \boldsymbol{\mu} + \boldsymbol{\gamma} G_t + \boldsymbol{\varepsilon}_t, \quad (1)$$

$$\boldsymbol{\varepsilon}_t = \sqrt{G_t}\mathbf{H}_t^{1/2}\mathbf{Z}_t, \quad (2)$$

where $G_t \sim \text{GIG}(\lambda, \chi, \psi)$ univariate i.i.d., $\mathbf{Z}_t \sim \mathcal{N}_k(\mathbf{0}, \mathbf{I}_k)$ i.i.d., $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_K)' \in \mathbb{R}^K$, $\boldsymbol{\gamma} = (\gamma_1, \ldots, \gamma_K)' \in \mathbb{R}^K$ and $\mathbf{H}_t \in \mathbb{R}^{K \times K}$ a positive definite, symmetric, conditional dispersion matrix of the form

$$\mathbf{H}_t = \mathbf{P}\mathbf{\Xi}_t\mathbf{P}', \quad (3)$$

with $\mathbf{P}$ *orthogonal* matrix of eigenvectors and $\mathbf{\Xi}_t$ the diagonal matrix of eigenvalues $\mathbf{\Xi}_t = \text{diag}\left(\xi_{1,t}, \ldots, \xi_{K,t}\right)$. The unique positive semidefinite square root of $\mathbf{H}_t$ is

$$\mathbf{H}_t^{1/2} = \mathbf{P}\text{diag}\left(\sqrt{\xi_{1,t}}, \ldots, \sqrt{\xi_{K,t}}\right)\mathbf{P}' = \mathbf{P}\mathbf{\Xi}_t^{1/2}\mathbf{P}', \quad (4)$$

i.e., the conditional return distribution has representation

$$\mathbf{Y}_t|\mathbf{\Phi}_{t-1} \stackrel{d}{=} \boldsymbol{\mu} + \boldsymbol{\gamma} G_t + \sqrt{G_t}\mathbf{P}\mathbf{\Xi}_t^{1/2}\mathbf{P}'\mathbf{Z}_t. \quad (5)$$

The COMFORT-PCA Model (2/4)

Definition: The COMFORT-PCA Model [Paolella et al., 2019] (continued)

For $\xi_{k,t}, k = 1, \ldots, K$ eigenvalues sorted by decreasing order only subset of eigenvalues is modeled as time-varying by assuming GARCH(1,1) dynamics for l largest eigenvalues, $1 \leq l \leq K$, i.e.

$$\xi_{k,t} = \omega_k + \alpha_k \epsilon_{k,t-1}^2 + \beta_k \xi_{k,t-1}, \text{ for } k = 1, \ldots, l, \tag{6}$$

where $\boldsymbol{\varepsilon}_t = \mathbf{Y}_t - \boldsymbol{\mu} - \boldsymbol{\gamma} G_t$ are de-meaned returns, $\boldsymbol{\epsilon}_t = \mathbf{P}'\boldsymbol{\varepsilon}_t$ their principal components. Remaining $K - l$ eigenvalues assumed time-invariant

$$\xi_{k,t} = \xi_k, \text{ for } k = l + 1, \ldots, K. \tag{7}$$

New model is called *COMFORT-PCA*: As in the COMFORT model, the *conditional* return distributions is modeled with MGHyp; but now PCA used for the eigendecomposition to identify the driving components of covariance dynamics

The COMFORT-PCA Model (3/4)

Remarks:

- Allows dynamics of covariance matrix to be driven by (small) set of statistical factors; allows for idiosyncratic shocks not captured by these factors
- We set no eigenvalues equal to zero (no dimension reduction), dispersion matrix $\mathbf{H}_t = \mathbf{P}\boldsymbol{\Xi}_t\mathbf{P}'$ required invertible for estimation algorithm (and portfolio optimization)
- Complexity reduction by reducing the number of cond. heteroscedastic factors. Same idea already in [Lanne and Saikkonen, 2007] (these authors set constant eigenvalues equal to one); show this allows to interpret their model (and ours) as a factor GARCH model in which cond. heteroscedasticity is due to l common factors
- Conditional distribution of $\mathbf{Y}_t$ given realization $G_t = g$ of mixing variable is multivariate Gaussian with mean vector $\boldsymbol{\mu} + \boldsymbol{\gamma} g$ and covariance matrix $g\mathbf{H}_t$; without conditioning on G_t the matrix $\mathbf{H}_t$ is not the covariance matrix, hence $\mathbf{H}_t$ called conditional *dispersion* matrix

Remarks (continued):

- Normalized (Gaussianized) de-meaned returns $\boldsymbol{e}_t = (\mathbf{y}_t - \boldsymbol{\mu} - \boldsymbol{\gamma} g_t)/\sqrt{g_t}$ follow multivariate Gaussian distribution with mean $\mathbf{0}$ and covariance matrix $\mathbf{H}_t$
- Robustness: PCA decomposition is not applied to covariance matrix of (possibly heavy-tailed) returns; instead to covariance matrix of $\boldsymbol{e}_t$ which is estimated by

$$\widehat{\Sigma}_{\widehat{\boldsymbol{e}}} = \frac{1}{T} \sum_{t=1}^{T} \widehat{\boldsymbol{e}}_t \widehat{\boldsymbol{e}}_t', \quad \text{where} \quad \widehat{\boldsymbol{e}}_t = (\mathbf{y}_t - \widehat{\boldsymbol{\mu}} - \widehat{\boldsymbol{\gamma}} \hat{g}_t)/\sqrt{\hat{g}_t}$$

  $\Rightarrow$ Our method can be regarded as "robust" PCA (resulting eigenvalues less sensitive to outliers in return series)

- Mean-variance mixture of MGHyp lends itself to estimation via EM-algorithm, see [Dempster et al., 1977], [Embrechts et al., 2015] or [Paolella and Polak, 2015a]

Notation: Let $\boldsymbol{\theta}_{\mathbf{M}} = [\boldsymbol{\mu}', \boldsymbol{\gamma}']'$, $\boldsymbol{\theta}_{\mathbf{GE}} = [\omega_1, \ldots, \omega_l, \alpha_1, \ldots, \alpha_l, \beta_1, \ldots, \beta_l]'$, $\boldsymbol{\theta}_{\mathbf{CE}} = [\mathrm{vec}(\mathbf{P}), \xi_{l+1}, \ldots, \xi_K]$, $\boldsymbol{\theta}_{\mathbf{D}} = [\lambda, \chi, \psi]'$, and $\boldsymbol{\theta} = [\boldsymbol{\theta}_{\mathbf{M}}, \boldsymbol{\theta}_{\mathbf{GE}}, \boldsymbol{\theta}_{\mathbf{CE}}, \boldsymbol{\theta}_{\mathbf{D}}]'$.

Complete log-likelihood: $\log L_{\mathbf{Y},G}(\boldsymbol{\theta}) = \log L_{\mathbf{Y}|G}(\boldsymbol{\theta}_{\mathbf{M}}, \boldsymbol{\theta}_{\mathbf{GE}}, \boldsymbol{\theta}_{\mathbf{CE}}) + \log L_G(\boldsymbol{\theta}_{\mathbf{D}})$, where $\log L_{\mathbf{Y}|G}$ is multivariate Gaussian log-likelihood. Further splitting

$$\log L_{\mathbf{Y},G}(\boldsymbol{\theta}) = \log L_{\mathbf{Y}|G}(\boldsymbol{\theta}_{\mathbf{M}}) + \log L_{\mathbf{Y}|G}(\boldsymbol{\theta}_{\mathbf{GE}}, \boldsymbol{\theta}_{\mathbf{CE}}) + \log L_G(\boldsymbol{\theta}_{\mathbf{D}}), \tag{8}$$

with

$$\log L_{\mathbf{Y}|G}(\boldsymbol{\theta}_{\mathbf{M}}) = -\frac{1}{2} \sum_{k=1}^{K} \sum_{t=1}^{T} e_{k,t}^2, \tag{9}$$

and

$$\log L_{\mathbf{Y}|G}(\boldsymbol{\theta}_{\mathbf{GE}}, \boldsymbol{\theta}_{\mathbf{CE}}) = -\frac{1}{2} \sum_{k=1}^{l} \sum_{t=1}^{T} \left(\log(2\pi g_t) + \log(\xi_{k,t}) + g_t^{-1} \frac{\epsilon_{k,t}^2}{\xi_{k,t}} - e_{k,t}^2 \right) \tag{10}$$

$$- \frac{1}{2} \sum_{k=l+1}^{K} \sum_{t=1}^{T} \left(\log(2\pi g_t) + \log(\xi_k) + g_t^{-1} \frac{\epsilon_{k,t}^2}{\xi_k} - e_{k,t}^2 \right). \tag{11}$$

E-step: Calculate expectation of G_t given $\mathbf{\Phi}_t$ and parameter estimates $\widehat{\boldsymbol{\theta}}$, where

$$G_t \mid \mathbf{\Phi}_t, \widehat{\boldsymbol{\theta}} \sim \mathrm{GIG}\left(\widehat{\lambda}^*, \widehat{\chi}_t^*, \widehat{\psi}_t^*\right), \tag{12}$$

with $\widehat{\lambda}^* = \widehat{\lambda} - K/2$, $\widehat{\chi}_t^* = \widehat{\chi} + (\mathbf{y}_t - \widehat{\boldsymbol{\mu}})' \, \widehat{\mathbf{P}} \widehat{\mathbf{\Xi}}_t^{-1} \widehat{\mathbf{P}}' \, (\mathbf{y}_t - \widehat{\boldsymbol{\mu}})$ and $\widehat{\psi}_t^* = \widehat{\psi} + \widehat{\boldsymbol{\gamma}}' \widehat{\mathbf{P}} \widehat{\mathbf{\Xi}}_t^{-1} \widehat{\mathbf{P}}' \widehat{\boldsymbol{\gamma}}$, see [Paolella, 2007], [Paolella, 2015]. Then use the moment formula for GIG [Paolella, 2007, Eq.9.18] to computed filtered values $\hat{g}_t$ and $\hat{g}_t^{-1}$ of the latent mixing realizations g_t and g_t^{-1}.

CM1-step: Update $\widehat{\boldsymbol{\theta}}_{\mathbf{M}}, \widehat{\boldsymbol{\theta}}_{\mathbf{CE}}, \widehat{\boldsymbol{\theta}}_{\mathbf{GE}}$ consecutively:

(a) Estimate $\widehat{\boldsymbol{\theta}}_{\mathbf{M}}$ by computing

$$\widehat{\boldsymbol{\theta}}_{\mathbf{M}} = \arg\max_{\boldsymbol{\theta}_{\mathbf{M}}} \log L_{\mathbf{Y}|\mathbf{G}}(\boldsymbol{\theta}_{\mathbf{M}}). \tag{13}$$

Done in closed form by weighted least squares: let $\boldsymbol{X} = [\mathbf{1}, \hat{\mathbf{g}}]$ and $\hat{\mathbf{g}}$ stacked vector of filtered $\hat{g}_t$. Further $\mathbf{W} = \mathrm{diag}(\hat{g}_1^{-1}, \ldots, \hat{g}_T^{-1})$ and $\boldsymbol{Y} \in \mathbb{R}^{T \times K}$ matrix of stacked return vectors, then WLS estimator for $\boldsymbol{\beta} = [\boldsymbol{\mu}, \boldsymbol{\gamma}]' \in \mathbb{R}^{2 \times K}$ is given by

$$\widehat{\boldsymbol{\beta}} = \left(\boldsymbol{X}' \mathbf{W} \boldsymbol{X}\right)^{-1} \boldsymbol{X}' \mathbf{W} \boldsymbol{Y}. \tag{14}$$

Estimation via ECME Algorithm (3/3)

(b) To compute $\widehat{\boldsymbol{\theta}}_{\mathbf{CE}}$ perform eigendecomposition of

$$\widehat{\Sigma}_{\widehat{e}} = \frac{1}{T}\sum_{t=1}^{T}\widehat{\boldsymbol{e}}_t\widehat{\boldsymbol{e}}_t', \tag{15}$$

where $\widehat{\boldsymbol{e}}_t = (\mathbf{y}_t - \widehat{\boldsymbol{\mu}} - \widehat{\gamma}\hat{g}_t)/\sqrt{\hat{g}_t}$. Store eigenvectors in matrix $\widehat{\mathbf{P}}$ and constant eigenvalues $\hat{\xi}_k$, $k = l+1,\ldots,K$, thus have $\widehat{\boldsymbol{\theta}}_{\mathbf{CE}} = [\mathrm{vec}(\widehat{\mathbf{P}}), \hat{\xi}_{l+1},\ldots,\hat{\xi}_K]$.
Note: number l of dynamic eigenvalues is chosen a priori (tuning parameter, see below)

(c) Compute $\widehat{\boldsymbol{\theta}}_{\mathbf{GE}}$ by maximizing GARCH terms in (10) given estimates $\widehat{\boldsymbol{\theta}}_{\mathbf{CE}}$, i.e.

$$\widehat{\boldsymbol{\theta}}_{\mathbf{GE}} = \arg\max_{\boldsymbol{\theta}_{\mathbf{V}}} \log L_{\mathbf{Y}|G}(\boldsymbol{\theta}_{\mathbf{GE}}, \widehat{\boldsymbol{\theta}}_{\mathbf{CE}}). \tag{16}$$

For this compute estimated principal components $\widehat{\boldsymbol{\epsilon}}_t = \widehat{\mathbf{P}}'\widehat{\varepsilon}_t$ (corresponds to data rotation in classical PCA). Solve this by parallel estimation of l independent univariate GARCH.

CM2-step: Given $\widehat{\boldsymbol{\theta}}_{\mathbf{M}}, \widehat{\boldsymbol{\theta}}_{\mathbf{SD}}, \widehat{\boldsymbol{\theta}}_{\mathbf{V}}$ from the CM1-step above, update $\widehat{\boldsymbol{\theta}}_{\mathbf{D}}$ by maximizing not the complete but incomplete log-likelihood ("*either*"), i.e. compute

$$\widehat{\boldsymbol{\theta}}_{\mathbf{D}} = \arg\max_{\boldsymbol{\theta}_{\mathbf{D}}} \log L_{\mathbf{Y}}\left(\boldsymbol{\theta}_{\mathbf{D}} \mid \widehat{\boldsymbol{\theta}}_{\mathbf{M}}, \widehat{\boldsymbol{\theta}}_{\mathbf{SD}}, \widehat{\boldsymbol{\theta}}_{\mathbf{V}}\right). \tag{17}$$

Iterate these E-, CM1 and CM2-steps until convergence.

Empirical Application: Absorption Ratio Modeling (1/4)

Data: daily (log)-returns from DJIA stocks (K=30), 02.01.1990-30.12.2016; rolling-windows size of $d = 1000$ days and $d = 180$ days

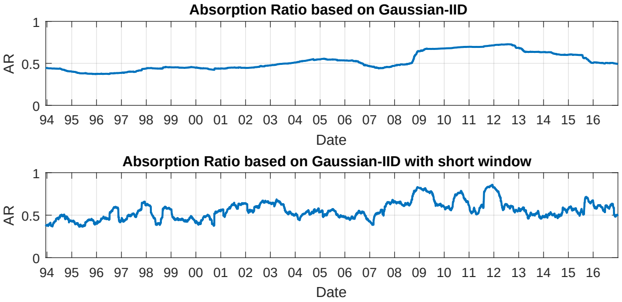
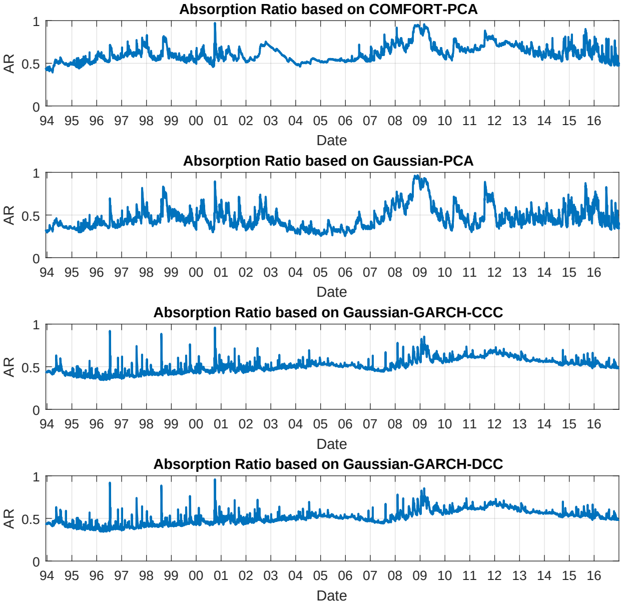


Figure: One-day ahead Absorption Ratio forecasts; based on PCA of covariance matrix

The basic AR is highly dependent on the length of the lookback period: barely any dynamics when too long; sensitive and erratic fluctuations when too short.

Empirical Application: Absorption Ratio Modeling and Forecasting (2/4)



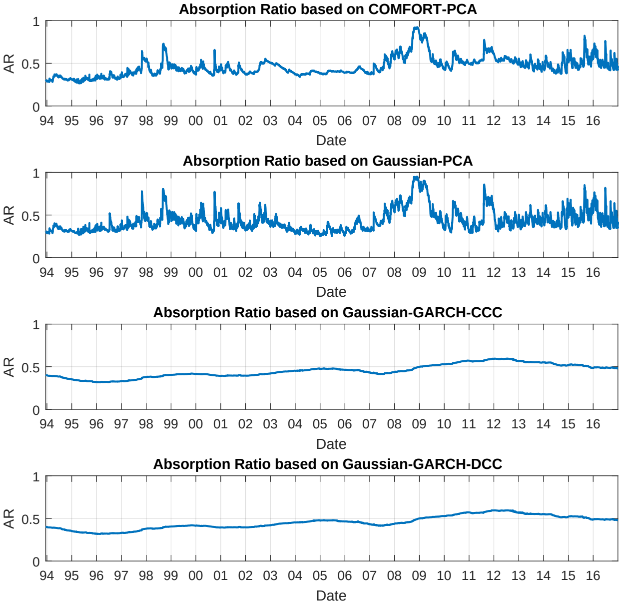
Empirical Application: Absorption Ratio Modeling (3/4)

- AR dynamics from Gaussian OGARCH and COMFORT-PCA much more realistic due to direct modeling of eigenvalues and thereby AR dynamics via univariate GARCH.
- Gaussian OGARCH leads to overly erratic AR, sensitive to small market shocks
- AR dynamics from GARCH-CCC and -DCC less intuitive: values in 1990s higher than in financial crisis

Fundamental critique of AR as systemic risk measure: Due to covariance matrix input the AR is largely driven by general level of market volatility, i.e., information content over pure market volatility is small.
$\Rightarrow$ Compute AR from PCA of correlation instead of covariance matrix

- AR forecasts based on correlation matrix from GARCH-CCC and -DCC barely move and lack any information about systemic risk level. DCC correlations appear to be inflexible when the number of assets is not small!
- AR forecasts based on correlation matrix from OGARCH and COMFORT-PCA appear much more suitable; able to detect the Asia crisis, Rubel crisis, Dotcom bubble, financial crisis as well as the Euro crisis.
- The MGHyp-based model more robust and delivers smooth forecast of systemic risk levels.

Empirical Application: Absorption Ratio Modeling and Forecasting (4/4)



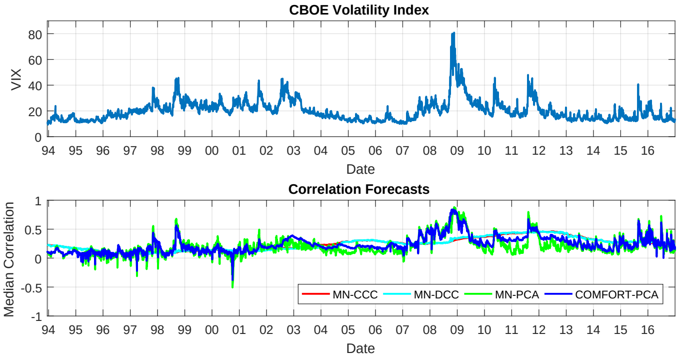
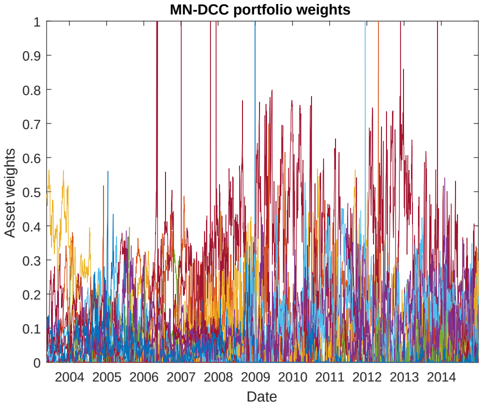


Figure: Median one-day ahead correlation forecasts

The dynamics of the correlation forecasts of the COMFORT-PCA and MN-OGARCH show well-known effect of increasing correlations during market distress. GARCH-CCC and -DCC delivers similar median correlations which underreact the market dynamics.

Evolution of portfolio weights for rolling window exercise of MN-DCC model
(min-var portfolio with daily rebalancing, trading period May 2003 - December 2014)



MN-DCC: Sharpe ratio 0.6053, max. drawdown 32.20%, average daily turnover: 22.82%
1/*N*: Sharpe ratio 0.5949, max. drawdown 42.11%,average daily turnover: 0.30%

Further Applications: Reducing Portfolio Turnover (2/7)

Using MGARCH models, such as CCC, DCC, VC, RSDC, for the covariance matrix estimate in portfolio optimization imply massive **turnover** and **transaction costs**.

MGARCH covariance dynamic better approximate the true DGP and often translate into better portfolio performance (without fees) but at cost of vastly increased rebalancing due to sensitivity of the optimal portfolio w.r.t covariance parameters

$\Rightarrow$ **MGARCH often infeasible in practice due to excessive portfolio rebalancing!**
Various approaches to reduce rebalancing and transaction costs:

1. Shrinkage of covariance matrix towards constant target
2. Regularization of objective function with rebalancing penalty term
3. Ex-post shrinkage of portfolio weights towards target portfolio which is constant over time

$\Rightarrow$ **Want to tackle problem from different angle: Employ COMFORT-PCA model with reduced number of dynamic eigenvalues to restrict covariance fluctuation to most relevant factors to reduce portfolio turnover**

Data set: daily stock returns from DJ30, 02.01.1990 - 30.12.2016, rolling windows exercise with $n = 1250$ data points and $T = 5556$ OOS steps.

MGHyp distribution difficult to fit due to flat likelihood in GIG parameters (λ, χ, ψ); hence consider two special cases: multivariate Student-t (semi-heavy-tailed) and Laplace distribution (thin-tailed)

Portfolio optimization methodology: min-ES strategy, for each rolling window portfolio weights computed from the one-day-ahead density prediction; using convex optimization formulation of [**?**], [**?**]. Results for min-variance strategy similar

Portfolio Turnover:

$$\mathrm{TO} = \frac{1}{T} \sum_{t=1}^{T} \sum_{d=1}^{K} |w_{d,t+1} - w_{d,t^+}|, \tag{18}$$

with w_{d,t^+} the proportion of wealth held in asset d at time $t+1$ just before rebalancing

$$w_{k,t^+} = \frac{\alpha_{k,t} P_{k,t+1}}{\sum_{k=1}^{d} \alpha_{k,t} P_{k,t+1}}, \tag{19}$$

where $\alpha_{k,t}$ is amount of stocks k corresponding to weight $w_{k,t}$, and $P_{k,t+1}$ is price of asset k at time $t+1$.

Proportional Transaction Costs: Let $\kappa > 0$ quantify level of proportional transaction cost (e.g. $k = 0.001$ or $k = 0.005$), then returns net of transaction costs are

$$(100 + R^N_{P,t+1|t}) = \left(1 - \kappa \sum_{d=1}^{K} |w_{d,t+1} - w_{d,t^+}|\right)(100 + R^G_{P,t+1|t}), \tag{20}$$

Influence of number l of leading eigenvalues with GARCH dynamics:

l	Daily Return	Volatility	Total Return	Max. Drawdown	Avg. Turnover	Sharpe	Sortino	Starr($ES_{99\%}$)
30	0.0384	**0.9425**	213.5774	21.2207	6.4834	0.6476	0.9292	0.0106
25	0.0382	**0.9425**	211.9708	21.5866	5.5432	0.6430	0.9237	0.0106
20	0.0386	0.9468	214.4396	22.3996	4.4133	0.6472	0.9302	0.0106
15	**0.0394**	0.9474	**218.8961**	21.5303	3.7944	**0.6603**	**0.9509**	**0.0109**
10	0.0391	0.9452	217.3208	20.3243	2.8667	0.6570	0.9452	0.0108
9	0.0383	0.9460	212.5288	20.6456	2.8204	0.6420	0.9232	0.0106
8	0.0384	0.9455	213.0744	20.9306	2.7845	0.6440	0.9263	0.0106
7	0.0383	0.9454	212.8472	20.7989	2.6817	0.6434	0.9252	0.0106
6	0.0382	0.9455	211.9914	20.6813	2.4963	0.6407	0.9211	0.0105
5	0.0383	0.9460	212.7041	20.7174	2.3875	0.6425	0.9238	0.0106
4	0.0385	0.9456	213.9012	20.2677	2.2695	0.6464	0.9296	0.0106
3	0.0388	0.9449	215.5154	20.2202	2.1920	0.6518	0.9375	0.0107
2	0.0390	0.9446	216.4007	**20.1937**	2.3219	0.6547	0.9416	0.0108
1	0.0385	0.9438	215.2387	20.3512	**1.9996**	0.6505	0.9376	0.0107
1/N	0.0439	1.1595	255.1055	27.5709	0.2501	0.6017	0.8538	0.0094

Table: Comparison of min-ES portfolios at 99%-ES, MLap-PCA with varying l

$\Rightarrow$ Turnover decreasing in l; without transaction costs max. SR attained for $l = 15$ (not $l = 30$), i.e. modeling GARCH to small eigenvalues detrimental

Sharpe ratios net of transaction cots of MLap-PCA model for varying l and κ:

l	*0bp*	*1bp*	*5bp*	*10bp*	*20bp*
30	0.6476	0.6367	0.5930	0.5384	0.4291
25	0.6430	0.6336	0.5963	0.5496	0.4562
20	0.6472	0.6398	0.6102	0.5732	0.4992
15	**0.6603**	**0.6539**	0.6285	0.5968	0.5332
10	0.6570	0.6522	0.6329	0.6088	0.5607
9	0.6420	0.6373	0.6184	0.5947	0.5473
8	0.6440	0.6393	0.6206	0.5973	0.5505
7	0.6434	0.6389	0.6209	0.5983	0.5533
6	0.6407	0.6365	0.6197	0.5988	0.5568
5	0.6425	0.6385	0.6225	0.6024	0.5623
4	0.6464	0.6426	0.6273	0.6083	0.5701
3	0.6518	0.6481	0.6334	0.6150	0.5781
2	0.6547	0.6508	**0.6351**	0.6156	0.5765
1	0.6505	0.6482	0.6350	**0.6182**	0.5845
1/N	0.6017	0.6012	0.5995	0.5974	**0.5931**

Table: Sharpe ratios net of transaction costs, min-ES portfolios at 99%-ES; MLap-PCA with varying l, under $\kappa = 0$, 1, 5, 10 and 20 basis point prop. transaction costs

$\Rightarrow$ Optimal l depends on level κ of transaction costs; for realistic levels of $3 - 5$ basis points $l = 3$ or $l = 2$ is optimal; only under extreme transaction costs is passive $1/N$ strategy superior!

Comparison of portfolio performance of various models without transaction costs:

Model	*Daily Return*	*Volatility*	*Total Return*	*Max. Drawdown*	*Avg. Turnover*	*Sharpe*	*Sortino*	*Starr($ES_{99\%}$)*
Mt-CCC	0.0431	**0.9162**	239.6066	**17.7935**	16.1237	**0.7473**	**1.0626**	**0.0121**
MLap-CCC	0.0432	0.9211	239.8098	19.3594	15.7349	0.7440	1.0563	0.0119
MAt-CCC	0.0416	0.9170	230.9833	20.9945	16.6828	0.7198	1.0209	0.0117
MALap-CCC	0.0413	0.9236	229.4366	23.7504	15.7016	0.7099	1.0052	0.0114
MN-DCC	0.0406	0.9254	225.6403	18.7262	22.5582	0.6968	0.9888	0.0112
MN-CCC	0.0402	0.9255	223.0400	18.7493	22.6754	0.6887	0.9777	0.0111
MN-RSDC	0.0396	0.9228	219.8195	20.3186	25.7596	0.6807	0.9645	0.0110
MLap-PCA	0.0388	0.9449	215.5154	20.2202	2.1920	0.6518	0.9375	0.0107
MALap-PCA	0.0386	0.9450	214.2952	20.1422	2.2730	0.6481	0.9320	0.0107
Mt-PCA	0.0383	0.9473	212.5364	21.2037	2.7326	0.6412	0.9223	0.0106
MAt-PCA	0.0376	0.9478	208.6284	21.1836	3.0937	0.6291	0.9046	0.0104
MALap-IID	0.0362	0.9230	201.2739	24.4040	2.1359	0.6231	0.8893	0.0103
MN-PCA	0.0342	0.9296	189.9685	29.1132	6.3259	0.5840	0.8302	0.0095
1/N	**0.0439**	1.1595	**255.1055**	27.5709	**0.2501**	0.6017	0.8538	0.0094

Table: min-ES portfolios at 99%-ES, all orthogonal COMFORT models are equipped with $l = 3$ eigenvalues having GARCH dynamics

$\Rightarrow$ Mt-CCC from COMFORT class has highest risk-adjusted returns, lowest drawdown and volatility, but turnover is rather high; Gaussian models have highest turnover; COMFORT-PCA easily outperforms MN-PCA

Sharpe ratios net of transaction cots for various models and levels of κ:

Model	*0bp*	*1bp*	*5bp*	*10bp*	*20bp*
Mt-CCC	**0.7473**	**0.7194**	0.6076	0.4678	0.1883
MLap-CCC	0.7440	0.7169	0.6083	0.4726	0.2013
MAt-CCC	0.7198	0.6909	0.5754	0.4309	0.1420
MALap-CCC	0.7099	0.6829	0.5749	0.4398	0.1699
MN-DCC	0.6968	0.6581	0.5033	0.3097	-0.0774
MN-CCC	0.6887	0.6498	0.4942	0.2996	-0.0893
MN-RSDC	0.6807	0.6364	0.4591	0.2374	-0.2057
MLap-PCA	0.6518	0.6481	**0.6334**	**0.6150**	0.5781
MALap-PCA	0.6481	0.6442	0.6290	0.6099	0.5717
Mt-PCA	0.6412	0.6366	0.6182	0.5953	0.5494
MAt-PCA	0.6291	0.6239	0.6032	0.5772	0.5254
MALap-IID	0.6231	0.6195	0.6048	0.5864	0.5497
MN-PCA	0.5840	0.5732	0.5299	0.4758	0.3676
1/N	0.6017	0.6012	0.5995	0.5974	**0.5931**

Table: Comparison of Sharpe ratios, min-ES portfolios at 99%-ES, all orthogonal COMFORT models are equipped with $l = 3$ eigenvalues having GARCH dynamics

$\Rightarrow$ Optimal model depends heavily on level of transaction costs; for realistic levels of transaction costs the COMFORT-PCA outperforms all models and the passive $1/N$ strategy; for extreme transaction costs all Gaussian models (except PCA) have negative Sharpe ratios due to huge turnover

- Presented new MGHyp-based OGARCH model called COMFORT-PCA: cond. return distribution accounts for excess kurtosis and asymmetry; a few leading eigenvalues equipped with GARCH(1,1) dynamics
- PCA decomposition not performed on covariance matrix of returns but of the normalized, de-meaned residuals, hence providing more robustness to outliers
- Model estimated via ECME-algorithm; PCA- and GARCH-estimation in each iteration
- Applications: Modeling and forecasting of systemic financial risks; correlations; portfolio optimization under transaction costs
- Absorption Ratio (AR) as popular, simple measure of systemic risk shows much improved dynamics when i.i.d. normal is replaced COMFORT-PCA model; MN-CCC and -DCC too inflexible and not robust to shocks; AR based on cond. correlations less affected by market risk (volatility)
- Optimal model depends on application: for AR modeling on the asset universe, for portfolio optimization on level of transaction fees


Thank You .

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