Structural Models of Corporate Bond Pricing with Maximum Likelihood Estimation*

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Abstract

Many empirical studies on the structural models of corporate bond pricing use a proxy to measure the market value of a firm's assets. This leads to the conclusion that barrier-independent models significantly underestimate corporate bond yields. Although barrier-dependent models tend to overestimate the yield on average, they generate a sizable degree of underestimation. This paper shows that a frequently used proxy for firm asset value is an upwardly biased estimator that makes the empirical framework work systematically against structural models of corporate bond pricing. When structural models are examined using the maximum likelihood estimation, we find substantial improvement in their performance. Moreover, both barrier-dependent and barrier-independent models tend to underestimate corporate bond yields on average. Our empirical analysis shows that structural models are useful for long-term and medium-term corporate bonds, but that improvement is needed for short-term bonds. We give suggestions for future model development.

Keywords: Corporate bond pricing, credit risk, maximum likelihood estimation, structural models, systematic bias.

1 Introduction

Structural models of corporate bond pricing originated with the seminal work of Black and Scholes (1973) and Merton (1974; henceforth the Merton model). By considering the capital structures of firms, the Merton model views equity as a call option on corporate assets, and views corporate debt as a default-free debt less a put option. However, this simple construction is inadequate to describe actual situations because it excludes the possibility of default before maturity, the effect of a stochastic interest rate, and the valuation of coupon-bearing bonds.

Extensions and refinements of structural models have been continuously made since the work of Merton. Black and Cox (1976) modeled the early default feature by introducing a default barrier. Geske (1977) viewed a corporate coupon bond as a portfolio of compound options. Longstaff and Schwartz (1995, LS) developed a simple framework that incorporates default barrier and stochastic interest rate to price corporate coupon bonds. Leland and Toft (1996, LT) derived the optimal capital structure to determine corporate bond value. Collin-Dufresne and Goldstein (2001, CDG) proposed a floating default barrier approach to model the target leverage ratio so that the error in pricing short term bonds with the LS model can be reduced.

Each model claims to be able to theoretically capture certain market phenomena, but it is important to contain empirical evidence with actual data. Jones, Mason and Rosenfeld (1984, JMR) were the first to empirically test the Merton model. Based on a sample of firms with simple capital structures and bond prices in the secondary market in 1977-1981, they showed that the predicted prices from the Merton model were too high, with a 4.52% on average, and that errors were more severe for non-investment grade bonds. Ogden (1987) conducted a similar empirical study with newly issued bonds and obtained a similar result. Lyden and Saraniti (2000) compared the performance of the Merton and LS models, and found that yield spreads were underestimated with the Merton model and that the LS model made no significant improvement.

Eom, Helwege and Huang (2004, EHH) recently conducted a comprehensive empirical study of structural models. They tested five structural models, including the Merton, Geske, LT, LS, and CDG models, taking into account the asset payout and stochastic interest rate. After carefully examining the capital structures of firms and characteristics of bonds, EHH obtained a sample of bonds from 1986-1997. To implement the structural models, they proxied the market value of corporate assets by the sum of the market value of equities and the book value of total liabilities, and used different ways to estimate the other model parameters. Within this setting, both the Merton and Geske models (barrier-independent models) underestimated yield spreads on average, whereas the LT, LS and CDG models (barrier-dependent models) tended to overestimate yield spreads on average, although there were also a sizable number of underestimations. Despite their empirical results, EHH proposed an extended Merton model for coupon bonds, and set some criteria for bond selection.

There are many different ways to implement structural credit risk models. The major concern is the estimation of the value and risk of a firm's assets, neither of which are directly observable. Ronn and Verma (1986) proposed a volatility restriction (VR) method to estimate firm value and volatility by simultaneously solving a pair of equations that match the observed stock prices and estimated stock volatility with model outputs. The VR method was adopted by the empirical works of JMR and Ogden (1987). EHH used the proxy for firm value but estimated the firm asset volatility by matching the estimated stock volatility with the model output. Therefore, it is a kind of mixed approach. Duan (1994) pointed out several theoretical problems of the VR method and derived a maximum likelihood (ML) approach. For commercial purposes, Moody's KMV uses the VR method to generate an initial guess of volatility, and then puts it into an iteration process to obtain a constant volatility in the stable stage of the process.

The implementation of structural models has gained attention in the literature recently because it affects the performance and testing of structural models. Ericsson and Reneby (2005) showed, by a series of simulations, that the ML approach obviously outperforms the VR method in terms of both lack of bias and efficiency. They claimed that, no matter how satisfactory the theoretical feature of a model, its empirical use may have been limited by the chosen implementation method. Duan et al. (2004) showed that the iteration process in the KMV approach is actually an EM algorithm for obtaining the maximum likelihood estimator, which means that it is equivalent to the ML approach of Duan (1994). However, to our knowledge, no empirical study of the structural models of corporate bond pricing has used the ML approach, and no academic work has been devoted to the comparison of the proxy and ML approaches.

This paper shows that structural models perform much better if the ML approach is used to estimate the model parameters, and, more importantly, that the proxy for firm value has many potential problems. When the proxy is carried forward to valuation, a significant pricing error is associated with all of the structural models of corporate bond pricing. In contrast, the pricing error is reduced significantly when the ML estimation is employed in an empirical study on the same set of market data. As the proxy for firm value essentially replaces the market value of debt with the book value of debt, our study is related to the paper of Sweeny, Warga and Winters (1997), which points out that empirical studies in which the market value of debt is replaced by the book value of debt distort the debt-equity ratio and cost of capital calculations.

Using option properties, we show that the proxy for firm value is an upwardly biased estimator. As the default probability decreases with firm value, the overstated firm value leads to the *underestimation* of corporate bond yields. Although the underestimation is less severe for barrier-dependent models, the error is still significant, and the natural question of why some empirical studies have found that barrier-dependent models *overestimate* corporate bond yields on average arises. We discover that the dividend yield as reported by Compustat, which is measured as the annual dividend divided by the end-of-year stock price, is an upwardly biased estimator. Consequently, the overstated dividend yields generate unreasonably high bond yields for firms that pay dividends. This effect is particularly pronounced in barrier-dependent models. Although the bias is limited to firms that pay dividends, the induced error is significant enough to make the predicted yields larger than the market yields on average. However, the underestimation of yields also occurs for firms that pay little or no dividends due to the proxy for firm value.

Before further strengthening our arguments with empirical data, we devise maximum likelihood (ML) estimations of the firm asset value and the asset value volatility for each model. For barrier independent models, we view equity as a call option on the firm, and employ the ML approach of Duan (1994) to estimate the parameters. For barrier-dependent models, we derive an ML estimation by viewing the market value of equity as a down-and-out call option on corporate assets. Our simulation verifies that the maximum likelihood estimators (MLE) are close to the true values, but that the proxy approach greatly overstates firm asset values and volatilities. The simulation shows that the proxy approach leads to an underestimation of corporate bond yields under both the Merton and LS models, whereas the MLE renders an accurate estimation.

The empirical data support our claims. We base our empirical study on the construction of EHH, including their criteria for the selection of corporate bonds, but our study is different in that the firm asset values and volatilities are obtained through ML estimation, the default barriers are set to the recovery value of debt, and the definition of stock dividend yield is revised so that it cannot exceed 100%. We test the extended Merton, LS, and CDG models. Our results show that the extended Merton model does not consistently underestimate corporate bond yields, and that the difference between the model and market yields is about -3 basis points on average. The LS model also generates realistic prices with an accompanying notable improvement on short-term and long-term bonds over the Merton model. Interestingly, the CDG model shows no improvement over the LS model, and performs the worst of the three models.

This paper contributes to the literature by recognizing the hidden bias in the proxy for firm value in structural models and by giving a bias-reduced empirical evaluation of the Merton, LS, and CDG models. We find that barrier-dependent models do not overestimate bond yields on average, but that all structural models under investigation tend to underestimate corporate bond yields. However, the LS model is significantly better than the Merton model, which suggests that ML estimation techniques is useful in the implementation of structural models.

The rest of the paper is organized as follows. Section 2 reviews the possible implementations of structural models, and discusses the strengths and weaknesses of each approach. Section 3 describes the ML estimation and verifies the estimation quality with a simulation. Section 4 reports our empirical framework and the results for the extended Merton, LS, and CDG models. Based on the empirical findings, we give suggestions for the future development of corporate bond pricing modeling. Conclusive remarks are presented in Section 5.

2 Implementation of structural models

A difficulty that arises when implementing structural models is the estimation of hidden variables, such as the value and risk of a firm's assets, asset payout ratio and default barriers. In this section, we first focus on different approaches to the estimation of the market value and volatility of a firm, and discuss the strengths and weaknesses of each approach. We then introduce our choice of asset payout ratios and default barriers with reasons for their selections.

2.1 The value and risk of a firm's assets

The simplest approach uses a proxy to measure the market value of a firm and then estimates the volatility through a time series of the proxy firm values. We call this estimation the pure proxy approach, which is actually the "Method I" used by JMR. In accounting principles, the market value of a firm's assets must be equal to the market value of equities plus the market value of debts. As the latter is not observable in the market, the proxy approach approximates it by using the book value of debts. Therefore, the proxy firm value, which is the sum of market value of equities and the book value of liabilities, changes over time through the fluctuation of equity values alone. This means that the firm asset volatility is estimated as the standard deviation of the returns of the proxy firm values. This approach does not depend on the particular features of a structural model and is typically easy to implement. However, the quality of estimation is unclear and may be detrimental to the performance of structural models.

To respect the features of structural models, Ronn and Varma (1986) proposed a volatility restriction (VR) method that obtains the firm value and volatility by solving a system of two equations. Specifically, for the Merton model the two equations are

$$S \;=\; C(V;\sigma_v) \text{ and } \sigma_e = \sigma_v \frac{V}{S}\frac{\partial S}{\partial V}, \tag{1}$$

where V and σ_v are the value and volatility of a firm, S and σ_e are the value and volatility of the equity, and $C(V;\sigma_v)$ is the call option pricing formula. In general, the first equation matches the observed equity prices with the prices of the model under investigation. The second equation restricts the estimated equity volatility to match the volatility that is generated by applying the Ito lemma to the equity pricing formula used in the first equation. Although the implementation is slightly more tedious than the pure proxy approach, the speed is very fast given modern computing power. At each point in time, this method produces a pair of estimates of firm value and volatility. Although the VR method violates the constant volatility assumption of most structural models, it is the most popular way of implementing structural models. Apart from academic research, Moody's KMV uses this approach in one part of the estimation process.

In between the foregoing two approaches is the mixed proxy approach that is used in the empirical study of EHH (2004). The market value of a firm is estimated as the proxy firm value, whereas a firm's volatility is calibrated to the second equation of the VR method. In this way, the estimation procedure is simpler than that of the VR approach but respects the model features through the second equation of (1). Similar to the pure proxy approach, the quality of estimation is not known.

The last estimation method we discuss is the maximum likelihood (ML) estimation proposed by Duan (1994). The idea of this method is to derive the likelihood function for the equity returns based on the assumptions that the firm value follows a geometric Brownian motion and the equity value is an option on the firm. By maximizing the likelihood function, parameters, such as the drift and volatility of a firm, are obtained. The firm asset value is then extracted out by equating the pricing formula to the observed equity price. This approach is theoretically sound as it is proven to be asymptotically unbiased and allows the confidence interval for the parameter estimates to be derived. The drawback of the ML approach is that it is a tedious and relatively time consuming approach, usually taking some ten seconds or longer to complete estimation with one sample path. However, most empirical studies involve the estimation of several thousand firms, and hence several thousand paths. One possible solution is to use several computers at once.

Duan (1994) pointed out several theoretical inconsistencies of the VR approach. Recently, interest in the implementation of structural models has been rekindled. Duan and Simonato (2002) applied ML estimation to deposit insurance value and showed that it outperforms the VR method with the Merton model. Ericsson and Reneby (2005) used a series of simulations to show that the ML approach of Duan (1994) clearly outperforms the VR method in parameter estimation for both barrier-independent and barrier-dependent models. To our knowledge, no work has been conducted on either the empirical analysis of structural models of corporate bond pricing with the ML estimation or the comparison of the ML estimation and the two proxy approaches.

As an empirical analysis with ML estimation is carried out in Section 4, we concentrate on the bias induced by the proxy firm value for the moment. We consider the effects of using the proxy firm value in the Merton and LS models, where the former is representative of barrier-independent models and the latter is representative of barrier-dependent models. We will show that the proxy for firm value is an upwardly biased estimator. Then, we will compare the pure and mixed proxy approaches.

2.1.1 The Merton model

In the work of Merton (1974), there is no intermediate default, and thus the terminal payoff of zero coupon bond holders takes the minimum of the face value of the bond (X) and the market value of assets (V_T). The current bond price ($B^M(V,X,T)$) is valued as a risk-free bond minus a put option ($P(V,X,T)$) on the current market value of assets (V) with a strike price X and a maturity T. Specifically,

$$B^M(V,X,T) = X \cdot D(T) - P(V,X,T), \tag{2}$$

where $D(T)$ denotes the default-free discount factor with a maturity T. However, the payoff for equity holders resembles the call option payoff with a strike price

X. Denote S as the market value of equities. We then have

$$S = C(V, X, T),$$

where $C(V, X, T)$ is the standard call option pricing formula.

Let V_{proxy} be the proxy firm value. The definition of the proxy then asserts that

$$V_{proxy} = S + X \text{ or, equivalently, } S = V_{proxy} - X.$$

By a property of standard call options, a call option premium must be greater than the intrinsic value, which implies that

$$C(V, X, T) = S = V_{proxy} - X < C(V_{proxy}, X, T).$$

As a call option is an increasing function of the underlying asset price, the foregoing inequality implies that the proxy firm value is an upwardly biased estimator. This overstated asset value causes the bond price of (2) to be overestimated and hence the yield spreads to be underestimated, which explains the significant underestimation of corporate bond yields with the Merton model in many empirical studies.

2.1.2 The LS model

Black and Cox (1976) introduced a failure barrier to trigger the default before debt maturity, which means that the market value of equities is viewed as a down-and-out call (DOC) option on the asset, whereas a zero coupon corporate bond is a portfolio of a long position in a risk-free debt, a long position in a down-and-in call, and a short position in a put option. However, the valuation of coupon bearing bonds is very sophisticated and has no analytical tractability.

The LS model is a simple model for coupon bearing bonds based on the work of Black and Cox (1976). As the LS model does not mention the market value of equities, we model it as a DOC option. Appendix A shows that the proxy firm value is an upwardly biased estimator if the default barrier is smaller than or equal to the debt level X, and that the bias decreases with the value of the barrier. We consider the default barrier in this range, because it is the usual assumption. A more detailed discussion of the choice of default barrier is given in the next subsection.

Under the LS model, a coupon bearing bond is decomposed into a sum of zero coupon bonds. This leads to the following pricing formula (see Appendix D).

$$B^{LS}(V,X,H,T) = \sum_{i=1}^{n} D(t_i) \cdot X_i \cdot [1 - \omega Q(V, X_i, H, t_i)], \tag{3}$$

$$\text{and} \quad X_1 = X_2 = ... = X_{n-1} = \frac{Xc}{2}, \quad X_n = X(1 + \frac{c}{2}),$$

where n is the total number of coupon paying dates $\{t_1, t_2, \cdots, t_n\}$ with $t_n = T$, and Q is the risk-neutral default probability. As the proxy for firm value is an upwardly biased estimator, the default probability $Q(V, X_i, H, t_i)$ becomes smaller than its true value for $i = 1, 2, \cdots, n$ if the proxy for firm value is used, and thus the bond yields are underestimated.

2.1.3 The pure proxy approach versus the mixed proxy approach

Both proxy approaches employ the same approximation of the firm value but differ in the estimation of a firm's volatility. We have just shown that both proxy approaches overestimate the firm value and hence underestimate the corporate bond yields. However, they may suffer from different degrees of underestimation due to the effect of the volatility estimate.

Consider a sample of n equally time-spaced observations of equity values $\{S_1, S_2, \cdots, S_n\}$ and a fixed book value of debt X over the period of observation. Both proxy approaches produce the same set of firm values $\{\hat{V}_1, \hat{V}_2, \cdots, \hat{V}_n\}$, where $\hat{V}_j = S_j + X$. The pure proxy approach measures the asset volatility as the sample standard deviation of the asset returns. That is,

$$\sigma^2_{pure} \Delta t \big|_{S_i} = \text{Var}\left(\frac{V_{i+1} - V_i}{V_i} \middle| S_i \right) = \text{Var}\left(\frac{S_{i+1} - S_i}{S_i} \frac{S_i}{S_i + X} \middle| S_i \right).$$

It is easy to see that the asset volatility obtained in this way is less than the stock volatility σ_e because $0 < S_i/(S_i + X) < 1$ for all $i = 1, 2, \cdots, n$. Moreover,

$$\sigma_{pure}|_{S_i} = \frac{S_i}{S_i + X} \times \sigma_e|_{S_i}.$$

For the mixed proxy approach, the volatility of the firm is estimated using the second equation of (1). By making the asset volatility the subject, we have

$$\sigma_{mix}|_{S_i} = \frac{S_i}{S_i + X} \left[\frac{\partial S}{\partial V} \right]^{-1}_{V = S_i + X} \times \sigma_e|_{S_i} = \left[\frac{\partial S}{\partial V} \right]^{-1}_{V = S_i + X} \times \sigma_{pure}|_{S_i}.$$

The quantity $\partial S/\partial V$, which is the delta of the standard call (down-and-out call) option for the Merton (LS) model, is always less than 1. This implies that the asset volatility of the mixed proxy approach is greater than that of the pure proxy approach.

A higher volatility leads to a high default risk of a firm and hence a higher credit yield spread. Therefore, the corporate bond yield predicted by the pure proxy approach is systematically less than that of the mixed proxy approach. In Section 3, we show by simulation that the mixed proxy approach underestimates corporate bond yield compared to the ML estimation. The underestimation should therefore be much more significant in the pure proxy approach.

Because of the shortcomings of the proxy firm value and the VR method, this paper examines the performance of structural models of corporate bond pricing with the ML estimation. To make the implementation possible, we specify the default barrier and asset payout ratio in the following way.

2.2 The default barrier

Usually, if not always, the default barrier is assumed to be less than or equal to the book value of liabilities, which is a reasonable assumption. When the asset-to-debt ratio is larger than 1, there is no incentive for a firm to declare bankruptcy or default on a loan, because the firm is still able to pay back loans by selling the asset to the market. In fact, it is not difficult to observe survival firms with a value that is much lower than the value of total debts.

However, there is no consensus of the exact position on the default barrier. Empirical studies that use a prudential barrier setting to the debt level include the works of Ogden (1987) and EHH. In the industry, Moody's KMV sets the default barrier to the default point, which is the short term debts plus a half of the long term debts, and is less than the total debt value. Wong and Choi (2005) empirically documented that default barriers tend to be less than the book value of total liabilities, and that 20% of the firms in their sample have a zero default barrier.

When implementing barrier-dependent models, we must specify a default barrier that is strictly positive but no greater than the book value of liabilities; otherwise, several inconsistencies may be encountered. For instance, a zero default barrier is inconsistent with our assumption of using a constant recovery rate. In our empirical study, we allow the default barrier to be the recovery value, which is the recovery rate time the book value of liabilities. When the firm value hits the barrier, it is assumed that the firm will declare bankruptcy and bond holders will receive the remaining value of the firm. This remaining value is actually the recovery value paid to bond holders if we assume a frictionless market and a strict priority rule. As all of the models considered in this paper are based on the assumption of no taxes and bankruptcy costs, this default barrier is a consistent choice. For a fair comparison, we also include the case of setting the barrier to the debt level as a control experiment.

2.3 The asset payout ratio

The comprehensive paper of EHH is the only empirical study of corporate bond pricing taking into account the asset payout ratio so far. This ratio captures the payout that the firm makes in form of dividend yield, share repurchase and bond coupons to equity holders and bond holders. The data of dividend yield and stock repurchase can be downloaded from Compustat.

All other things being fixed, the default probability increases with the asset payout ratio. As the firm value should move downward after a payout event, the probability that the firm will be unable to honor future obligations increases and thus this payout ratio plays a crucial role in the pricing of corporate bonds.

We use the spirit as EHH, but make several modifications to their approach. First, we do not directly use the reported dividend yields, but rather use a revised definition. The reported dividend yield from Compustat is calculated by dividing the annual dividends by the end-of-year stock price, which is an upwardly biased estimator. For firms that pay a large amount of dividends, the corresponding figures are usually in excess of 100%. For example, the reported dividend yields for the USG Corporation in 1988 and the Georgia Gulf Corporation in 1990 are 668% and 282%, respectively. These figures are misleading because the actual payout should not be that high; otherwise, an arbitrage profit can be made by purchasing the stock to receive dividends, the total value of which is greater than the initial investment.

An overstated dividend yield leads to an overestimation in asset payout ratio and hence corporate bond yields. When we take this effect together with the fact that the underestimation of corporate bond yields that is caused by the proxy firm value is less severe for barrier-dependent models, we have a potential explanation for the finding of past empirical studies that barrier-dependent models tend to overestimate corporate bond yields on average, but with a sizeable number of underestimations. In particular, underestimation occurs for low-dividend paying firms and overestimation occurs for firms that pay a large amount of dividends.

Strictly speaking, the reported dividend yield gives an erroneous perception to investors, because the equity price should be adjusted downward after a dividend payment, and the end-of-year stock is actually the price after the dividend. If this is used as a denominator to compute the dividend yield, then the number will be overestimated. Thus, we revise the definition to be the annual dividend over the sum of the end-of-year stock price and the annual dividend. This value is easy to obtain and is guaranteed to be less than or equal to 100%. Suppose that the reported dividend yield is $\hat{q}$. The revised dividend yield then becomes $q = \hat{q}/(1 + \hat{q})$.

Second, we recognize that the effects of the asset payout ratio are different for equity holders and bond holders. In other words, we use two different values of asset payout ratio in estimation and corporate bond pricing procedures. We now offer an explanation for this. For stock options, the option holder cannot receive dividends paid before an option's maturity, and thus the dividend yield in the call option pricing formula lowers the option price to account for this effect. However, the story is rather different for equity holders. When the asset payout is due to stock dividend, the payout amount is essentially given to equity holders, who experience no loss. Bond holders, however, suffer from a higher credit risk. The same concept applies to stock repurchases as the money goes to equity holders to buy back the stocks, and thus the effective asset payout ratio to equity holders should exclude the stock dividend and stock repurchase. The last component of the asset payout ratio is bond coupons. However, bond coupons are also not included in the effective asset payout ratio to equity holders in our estimation process because the book value of liabilities has already taken into account the coupon payment. Therefore, we set the effective asset payout ratio to equity holders to zero to avoid a double count of the bond coupon effect.

In the bond pricing procedure, we measure the asset payout ratio δ according to the revised dividend yield q and stock repurchases, but exclude bond coupons (see Section 4.2 for more information). The reason is that coupon values have been entered into the corporate bond pricing formula: if the coupon values are also included in the asset payout ratio, then the effect will be counted twice. In fact, the original papers of Merton, LS, and CDG use a zero asset payout ratio. In the later two papers, the effects of dividends and stock repurchases are abstracted from the analysis but the coupon effect is added. Therefore, we assume that the asset payout ratio to debt holders does not contain bond coupons.

In summary, the implementation of structural models with proxies can provide us with a distorted picture of the performance of the models. For barrier-independent models, the proxy for firm value is the dominated biased factor that leads to the underestimation of credit yield spreads. For barrier-dependent models, the effect of this factor is less severe, but still significant. However, the overstated asset-payout ratio may generate a bias to dividend paying firms, which results in the overestimation of credit yield spreads.

3 Maximum Likelihood Estimation

As the proxy for the market value of assets is biased upward, we adopt the maximum likelihood estimator (MLE) approach in this paper. For barrier-independent models, we use the approach of Duan (1994), and for barrier-dependent models, we view equity values as a down-and-out call option on the firm value to devise the corresponding MLE approach. This section provides the detail of the formulation and verifies the approach by means of a simulation.

3.1 The MLE approach for the Merton model

The parameters are the asset drift (μ) and asset volatility (σ). For the Merton model, Duan (1994) showed that the likelihood function for the equity return is

$$L(\mu,\sigma) = \sum_{i=2}^{n} \{\ln g(v_i|v_{i-1}) - \ln[V_i \cdot N(d_1)|_{V=V_i}]\}, \tag{4}$$

where $N(\cdot)$ is the cumulative distribution function for a standard normal random variable and V_i and v_i denote the asset price and the log of the asset price at time i, respectively. The explicit expressions of $g(\cdot)$ and d_1 are given in Appendix B, where we also present the detail formulation.

MLEs are parameters that maximize the likelihood function (4), subject to the constraints that the market values of equities are equal to the call option pricing formula, that is,

$$\max_{\mu,\sigma} L(\mu,\sigma) \quad s.t. \quad S(t_i) = C(t_i, V(t_i), \sigma),\ \forall\, i = 1, 2, \cdots, n.$$

Then, the firm values $V(t_i)$ are solved numerically from the call option formula using the value of σ.

3.2 The MLE approach for barrier-dependent models

For barrier-dependent models, the market value of equity is viewed as a DOC option, rather than a standard call option. The pricing formula of a DOC option is given in Appendix C. As the asset price should not go below the default barrier before bankruptcy occurs, the density function of the log-asset-price becomes [see Rubinstein and Reiner (1991)],

$$g^B(v_i|v_{i-1};\mu,\sigma) \quad = \quad \varphi(v_i - v_{i-1}) - e^{2(\eta-1)(h-v_{i-1})}\varphi(v_i + v_{i-1} - 2h), \quad (5)$$

where

$$h = \log H, \qquad \eta = \frac{\mu}{\sigma^2} + \frac{1}{2},$$

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi(t_i - t_{i-1})}} \exp\left\{-\frac{[x - (\mu - \sigma^2/2)\cdot(t_i - t_{i-1})]^2}{2\sigma^2(t_i - t_{i-1})}\right\}.$$

In our estimation process, the function $g^B(\cdot|\cdot)$ takes the form of (5) if the underlying asset value is larger than the barrier, and zero otherwise. Given the explicit formula of a DOC option, the option delta ($\Delta(V)$) is calculated by differentiating the pricing formula with respect to V. Following a similar procedure to that of the Merton model, we obtain the log-likelihood function as being

$$L^B(\mu,\sigma) = \sum_{i=2}^{n}\{\ln g^B(v_i|v_{i-1}) - \ln[V_i \cdot \Delta(V_i)|_{V=V_i})]\}. \quad (6)$$

We then estimate the parameters by solving the following optimisation problem.

$$\max_{\mu,\sigma} L^B(\mu,\sigma) \quad s.t. \quad S(t_i) = \text{DOC}(t_i, V(t_i), \sigma),\ \forall\, i = 1, 2, \cdots, n.$$

Finally, we obtain the firm value $V(t_i)$ inversely from the DOC pricing formula.

3.3 Survivorship consideration

In our empirical study, the sample is drawn from survival companies, which may lead to a survivorship bias in the estimations. We recognize that maximum likelihood estimation with survivorship has been considered by Duan et al. (2003), who found that the original approach of Duan (1994) leads to an upward bias in the asset drift, but that the other parameters are obtained with a high quality. However, the survivorship bias has no impact on the testing of corporate bond pricing models. Structural models value corporate bonds in the risk-neutral world in which asset drift is replaced by the risk-free interest rate. Thus, the biased drift value plays no role in corporate bond pricing formulas. The inclusion of the drift in the estimation procedure aims at enhancing the estimation quality of the volatility.

3.4 Simulation tests

Our simulation verifies the performance of the estimation scheme. In this simulation exercise, we use $r = 6.5\%$, $\mu = 8\%$, σ = 0.25, and an initial firm value of 1. One-year (260-day) sample paths are generated according to the Black-Scholes dynamics. Consider the debt maturities T of 2, 5, 10 and 20 years. The face value of the debt X takes three possible values, 0.3, 0.5 and 0.7, which represent the different leverage levels (or creditworthiness) of a company. To test the Merton model, we compute the market values of corporate equities by the standard call option formula. However, we use the DOC option pricing formula to calculate the market values of equities under the LS model.

Suppose that the extended Merton model introduced by EHH, see Appendix D, and LS model are correct models for two different economies. This simulation on the one hand, attempts to show that the proxy for firm value leads to an underestimation of corporate bond yields, and on the other hand is used to check the performance of the MLE. We directly compare the MLE approach with the mixed proxy approach, which produces less underestimation in corporate bond yields than the pure proxy approach. We first simulate equally time-spaced market values of the firm based on specified parameters, and then generate equity values and corporate bond prices using both models. These generated data are then regarded as market observable values. The detailed procedures of the MLE approach and the proxy approach are summarized as follows.

1. Extended Merton model with $K = X$ and $\omega = 0$.

   (a) MLE approach. The approach of Duan (1994) is employed to estimate the asset volatility and the market value of assets. By plugging the estimates back into the extended Merton model of corporate bond pricing, the predicted credit yield spreads are obtained.

   (b) Proxy approach. We estimate the market value of a firm's asset by the proxy for firm value. The asset value volatility, σ_v, is solved from the equation $\sigma_e = \sigma_v \frac{V_t}{S_t} \frac{\partial S_t}{\partial V_t}$, where σ_e is the equity volatility, and S_t and V_t denote the market value of equity and the proxy asset value at time t, respectively. These estimates are substituted into the extended Merton model to estimate credit yield spreads.

2. LS model with $H = X$ and $\omega = 51.31\%$.

   (a) MLE approach. We view the market value of equity as a DOC option and perform our proposed MLE approach to estimate the asset volatility and the market value of the firm. These estimated parameters are used to derive corporate bond yield using the LS model.

(b) Proxy approach: We use the same procedure as for the extended Merton model, except that the LS model is used this time.

Finally, we compare the credit yield spreads and bond prices that are obtained from the proxy and MLE approaches for each model.

The simulation results for the Merton and the LS models are respectively given in Table 1 and Table 2, and the percentage errors in prices, yields and yield spreads are reported. The percentage error in prices is the model prices minus the market price divided by the market price, where a positive number indicates an overestimation. The percentage errors in yields and yield spreads are calculated in the same manner.

3.4.1 Simulation results for the Merton model

Table 1 shows that the average percentage errors in prices and yields are all close to zero for the MLE approach, whereas using the proxy firm value the average percentage errors in prices are significantly positive and those in the yields and yield spreads are significantly negative. Panel A shows that the errors are more severe for zero coupon bonds, and Panel B implies that the errors are more pronounced for high leveraged firms. The percentage errors in yield spreads, shown in Panel C, are consistently less than -90% for all maturities. The simulation suggests that underestimations of bond yields with the Merton model are probably due to the hidden bias of the proxy for firm value.

We further illustrate our simulation result by graphs. In Figure 1, the circles represent the percentage errors in yields that are obtained from the MLE, and the crosses represent those from the proxy approach. Figure 1a contains the result for all credit quality and Figures 1b-d present the results for high, medium and low ratings respectively. The crosses are generally beneath the circles for all cases, which shows that the proxy for firm value leads to the underestimation of corporate bond yields.

3.4.2 Simulation results for the LS model

Table 2 summarizes the results of the LS model. It can be seen that the MLE approach definitely outperforms the proxy approach, and that the errors that are induced by the proxy are less significant than those of the Merton model. However, the errors are not negligible.

Figure 2 consists of four pictures. Figure 2a plots the percentage errors in the yields against the debt maturities for all bonds. We can see that the error points of the MLE are located around zero, whereas most of the points of the proxy approach are negative. Figures 2b, 2c and 2d show the percentage errors in yields for high, medium, and low rating bonds, respectively. We recognize that proxy always underestimates the yields of high and medium rating bonds. In Figure 2d, we can see that there are some points with positive percentage errors in the yields with the proxy approach, and thus the errors that are generated by the proxy are partially offset by the imposition of a default barrier for low rating bonds.

In summary, our simulation further supports that the proxy of firm value is inappropriate and leads to the underestimation of bond yields when all other parameters are fixed. This bias occurs in both barrier-dependent and barrier-independent models.

4 Empirical study

An empirical study is conducted to check whether the performance of structural bond pricing models is improved when the MLE approach is used, and whether the empirical results in the past are driven by the proxy for firm value. We also empirically examine the Merton, LS, and CDG models using the MLE approach.

4.1 Criteria of bond selection

Based on the criteria of EHH, we select bonds with simple capital structures and sufficient equity data. The bond prices on the last trading day of each December for the period 1986-1996 were obtained in the Fixed Income Database. We choose non-callable and non-putable bonds that are issued by industrial and transportation firms, and exclude bonds with matrix prices and those with maturities of less than one year. There are nearly 7,000 bonds that meet these criteria.

To have simple capital structures, we consider firms with only one or two public bonds, and sinkable and subordinated bonds are excluded. We examine the characteristics of the firms with information that was provided by the Rating Interactive of Moody's Investor Services. We regard firms with an organization type as corporations and exclude those with a non-US domicile. Firms in broad industries such as finance, real estate finance, public utility, insurance and banking are also excluded from our sample. At this stage, our sample consists of 2,033 bonds.

To measure the market value of corporate assets, we restrict ourselves to firms that have issued equity and provide regular financial statements. Therefore, we downloaded the market values of equities from Datastream and total liabilities and reported dividend yields from CompuStat for the period 1986 to 1996. By matching all of the available data and excluding some firms that were acquired, our sample ultimately contains 807 bonds issued by 171 firms.

The summary statistics of the data are exhibited in Table 3. Panel A shows that our sample contains bonds with maturities that range from one year to fifty years, with an average of ten years. This wide range of maturities enables us to study the maturity effect of different structural bond pricing models. Our sample covers zero coupon bonds and bonds with high coupon rates, at a maximum of 15%. The range of yield-to-maturity is wide, from 4% to 22.5%. The bonds in the sample fall within a large credit spectrum. Most bonds belong to the investment grade according to Moody's and S&P, and some are junk bonds. These large discrepancies in ratings allow us to check the performance of the structural models for different credit qualities. Our sample includes different sizes of firms that carry at least US$231 million of market capitalisation to a maximum of US$96 billion. The total liabilities of these firms range from US$114 million to a maximum of US$150 billion.

Panel B presents the mean of the time to maturity, coupon rates, yield-to-maturity, Moody's rating, S&P rating, market capitalizations, and total liabilities. The mean of the Moody's and S&P ratings are quite stable, but the values of time to maturity, coupon rate, and yield-to-maturity vary from 8 to 11.5, 7.55 to 9.75, and 6.4 to 10, respectively.

4.2 Parameters of the models

Firm-specific parameters include the market value of assets (V), asset volatility (σ), book value of liabilities (X), asset payout ratio (δ), and default barrier (H).

To make comparisons, we use both the proxy and the MLE approaches to estimate the market value of assets and the asset value volatility. Our proxy approach always refers to the mixed proxy approach discussed in Section 2. Although not reported, the pure proxy approach performs very poor. In the MLE framework, estimation is based on a one-year time series of market values of equities. We use the approach of Duan (1994) for the Merton model and the likelihood function of (6) for the barrier-dependent models, which include the LS and CDG models. We assume that a zero rebate is paid to equity holders upon default.

To refresh ideas, we recall that, for the proxy approach, the market value of corporate assets is measured by the sum of the market value of equities and the book value of total liabilities. The asset volatilities are obtained by the leverage $\sigma_e = \sigma_v \frac{V_t}{S_t} \frac{\partial S_t}{\partial V_t}$, where S_t and V_t denote the equity value and the proxy firm value at time t, respectively. Moreover, σ_v and σ_e denote the asset volatility and historical equity volatility respectively. The historical equity volatility is measured over a window of 150 trading days.

The asset payout ratio, δ, is the equity payout ratio times leverage. The equity payout ratio is the revised dividend yield for firms with no stock repurchases in the year. Let $\hat{q}$ be the reported dividend yield and D be the annual dividend. Then, the asset payout ratio for this case is given by

$$\delta = \frac{D}{S+D} \frac{S}{V} = \frac{\hat{q}}{1+\hat{q}} \frac{S}{V} = q \frac{S}{V}.$$

Otherwise, the asset payout ratio is calculated as

$$\delta = \frac{D + D_r}{S + D + D_r} \frac{S}{V},$$

where D_r is the total value of stock repurchase over a year. However, stock repurchase happens very rare compared to the stock dividend. Therefore, the effect of asset payout ratio is largely driven by dividends.

For the extended Merton model, we follow EHH and specify the default threshold K as the face value of total liabilities. For the LS model and CDG models, the (current) default barriers H are set to the recovery rate times the face value of the total liabilities, which is a consistent choice. As equity holders receive no rebate upon default, the recovery value that is received by bond holders should be equal to the value of the whole firm at the time of default. Barrier-dependent models assert that the firm value is at the barrier level upon default, but if bankruptcy costs are taken into account, the default barrier is expected to be higher. In this paper, we abstract the bankruptcy costs.

4.2.1 Interest rate parameters

The Merton model assumes a constant interest rate which we measure by the instantaneous interest rate fitted to the Nelson-Siegel (1987) model. The LS and CDG models employ stochastic interest rates following the Vasicek (1977) model. We calibrate the four parameters that are used in the Vasicek (1977) model to the yield data of constant maturity treasury bonds, the data of which were obtained from the Federal Reserve Board's H15 release.

Specifically, the Nelson-Siegel (1987) model estimates the yield of default-free bonds, y_{NS}, as

$$\hat{y}_{NS} = \beta_0 + \delta_1(\beta_1 + \beta_2)\frac{1 - e^{-\tau/\delta_1}}{\tau} - \beta_2 e^{-\tau/\delta_1},$$

where τ is the time to maturity. To calibrate the parameters, we search for the optimal values of $\beta_0, \beta_1, \beta_2,$ and δ_1 such that the sum-of-squared-error between the model yields and the market yields is minimized.

The Vasicek model is calibrated in the same way as the Nelson-Siegel model. The LS and CDG models require the correlation coefficient between the asset value returns and changes in the risk-free rate. As the market values of assets have been estimated either by MLE or the proxy approach, we directly calculate the correlation coefficient, ρ, between the asset returns and changes in interest rate.

4.2.2 Stationary leverage process parameters

The stationary leverage process parameters are required for the CDG model. We estimate two sets of parameters for both the MLE and the proxy approaches. By the asset price process of (D.2), the process of the log-default-barrier is given by (D.8). By an application of Ito's lemma, the process of the log-target-leverage-ratio, $\ln L_t = \ln(V_t/H_t)$

$$\begin{aligned} d\ln L_t &= [\mu + \lambda\bar{\nu} + \lambda(\phi r_t - \ln L_t)]\, dt + \sigma dW_{1t}, \\ \bar{\nu} &= (\nu - \phi\theta) - (\delta + \sigma^2/2)/\lambda. \end{aligned} \tag{7}$$

Given the value of the default barrier and the market values of assets, which are either estimated by the MLE or the proxy approach, the one-year time series of $\ln L_t$ are produced. We then search for the optimal values of $\lambda, \bar{\nu}, \phi$ by minimizing the sum-of-squared-error between the "observed" and predicted values of the log-target-leverage ratios, where the predicted values are calculated by equation (7).

4.2.3 Bond specific parameters

The coupon rate (c) and maturity (T) of the bonds are obtained from the Fixed Income Database, which enables us to derive the remaining coupon-paying days for each bond.

For the recovery rate (ω) of a bond, the paper by Altman and Kishore (1996) shows that the recovery rates for senior secured and senior unsecured debt are about 55% and 48%, respectively. Keenan, Shtogrin and Sobehart (1999) also find that the average bond recovery rate is around 51.31% of the face value of a bond. We follow EHH in taking a recovery rate of 51.31%. We apply this recovery rate to the extended Merton, LS, and CDG models.

4.3 Empirical results

The empirical results for the Merton, LS, and CDG models are summarized in Table 4-7, in which the percentage errors in prices, yields, and yield spreads are provided. The effects of agency ratings and bond maturities are reported in Table 6 and Table 7, respectively. We regard bonds with an S&P rating of A or above as high rating bonds, those with a BBB-rating as medium rating bonds and others as junk bonds. We regard bonds with a maturity of less than or equal to 5 years as short-term bonds, of 5-15 years as medium-term bonds, and others as long-term bonds.

4.3.1 The Merton model

Table 4 shows the performance of the Merton model. The average percentage errors in the prices and the yields are respectively 7.22% and -15.17% for the proxy approach, and for the MLE approach the errors in the prices and the yields are 2.37% and -1.82%, respectively. The MLE approach thus consistently improves the performance of the Merton model in predicting prices and yields.

A similar conclusion can be drawn for the yield spreads. In fact, the MLE approach produces an average prediction error in the yields of -3 basis points, whereas the proxy approach gives an error value of -126 basis points. This offers an empirical evidence that the proxy firm value makes the Merton model generate a sizable of underestimation of yields. One may recognize that the standard deviations of our MLE approach are greater than those of the proxy approach. We stress that a small standard deviation together with a wrong mean value indicates a serious bias.

Figure 3 plots the errors in the yields against the bond maturities. Figure 3b shows the performance of the proxy approach, in which most points fall into the negative region. Figure 3a shows the empirical results of the MLE approach. We observe that most points are crowded near zero, which provides evidence that the proxy for firm value leads to the underestimation of bond yields. Moreover, the

MLE approach offers a better estimation of corporate bonds, with quite a number of outliers in the set of short-term bonds. Although the Merton model underestimates corporate bond yields on average, it does not consistently underestimate the yields, as Figure 3a shows that there are many points in the positive region.

Using the MLE approach, the Merton model, shown in Figure 3, can seriously overestimate yields for short-term bonds. This is not surprising, because the Merton model produces unrealistically high yields for short-term, but not very short-term, bonds. The Merton model, shown in Panel A of Table 6, tends to underestimate the yields for high and medium ratings and significantly overestimates yields for low ratings. When we check our database, we find that most junk bonds have short maturities. Therefore, the result may be driven by the maturity effect, rather than the effect of ratings. From Panel A of Table 6, we can see that the Merton model overestimates short-term bond yields, underestimates long-term bond yields, and performs the best for medium-term bonds.

4.3.2 The LS model

For the LS model, Table 4 shows that the MLE approach again outperforms the proxy approach, and that the proxy for firm value leads to the underestimation of bond yields. The average percentage error in the prices for the proxy approach is 6.19%, which is significantly positive, whereas the average percentage error in the yield is -9.45%, which is significantly negative. Adopting the MLE approach, the average percentage errors in the prices and yields are 3.57% and -4.38%, respectively, and are relatively small in magnitude. The proxy approach considered in this paper does not include the reported dividend yield but uses the revised version. Therefore, the overestimation in corporate yield with barrier-dependent models in the past empirical studies may be due to the dividend yields reported by Compustat since the overestimation does not occur in either our proxy or MLE approaches.

To further examine the effect of the reported dividend yields, we carry out the empirical analysis again by setting the default barrier to the book value of liabilities to rule out the effect of our choice of default barrier. Table 5 shows that the proxies for firm value and the default barrier together do not overestimate bond yields in general, and therefore the overestimation in yield is the consequence of using the reported dividend yield. Interestingly, when the MLE approach is used with barrier setting to the book value of liabilities, the average bond yield is overestimated even with the revised dividend yield. However, when we compare Table 5, Panel B of Table 6 and Panel B of Table 7, we can seen that the performance of the MLE approach with the barrier set to the recovery value is much better than the performance of the approach with barrier at the total debt level. This suggests that setting the default barrier to the recovery value is more appropriate.

Figure 4 plots the difference between the model and market yields against bond maturity. The LS model (Figure 4a) tends to overestimates yields when default barrier is set to the book value of liabilities, but it tends to underestimate yields (Figure 4b) when the default barrier is set to the recovery value. This implies that there is a default barrier between the two values such that the average percentage error in the yields is zero. This is a reasonable observation, as the bankruptcy costs, which we ignore, should pull up the barrier to higher than the recovery value. Future research should consider the effect of bankruptcy costs.

Figure 4c presents the results for the proxy approach with the default barrier set to the recovery value, in which most of the points are located in the negative region. A comparison to Figure 4b and 4c shows that the underestimation of yields is less severe with the MLE approach, which suggests that the MLE approach improves the predictive power of the LS model.

Like the Merton model, there is evidence, as shown in Figure 4b, of both extreme underestimation of yields and extreme overestimation, but the problem is much less severe here. Extreme overestimation often appears for the short-term bonds. Table 7 reveals that the LS model does better on short- and long-term bonds than the Merton model. The percentage error in the yields of short-term bonds is -5.88%, the magnitude of which is smaller than that of the Merton model (6.78%). The improvement is obvious in the case of long-term bonds, as the LS model generates a percentage error of -1.44% against that of the Merton model of -7.34%.

Table 6 shows that the LS model outperforms the Merton model for both high and low rating bonds. The percentage errors in the yields of low ratings in the LS and Merton models are -9.23% and 40.13%, respectively. Although the percentage error in high ratings for both models are similar, the LS model is much less volatile. The standard deviation of the percentage errors in the yields is almost half of that of the Merton model. Therefore, the imposition of a default barrier improves structural models, as it captures the effect of early default. This improvement is particularly pronounced for both long-term bonds and low rating bonds. However, the LS model is still inadequate in describing the credit risk of short-term bonds. To reduce the extreme overestimation of short-term bond yields, it may be useful to consider a floating barrier model.

4.3.3 The CDG model

Table 4 contains the overall performance of the CDG model. We observe that the MLE approach and the proxy approach have a similar performance. The average percentage errors in the prices and yields for the proxy approach are 5.40% and -10.83%, respectively, whearas those for the MLE approach are 5.71% and -12.19%, respectively. None of the approaches generates a positive average percentage error in the yields. From Panel C of Table 4, we can see that corporate yields are overestimated on average by the CDG model, and thus this bias again arises from the reported dividend yields.

In Figure 5, there is no obvious difference between the MLE approach and the proxy approach in the CDG model, and most of the points are located in the negative region for both the MLE and the proxy approach. This suggests that the MLE approach does not improve the performance of CDG model.

There are two possible reasons for this. First, the CDG model does not mention about the modeling of equity value, and as this model involves a default barrier, we use the DOC option framework as a proxy to model equity. However, the CDG model is based on the leverage ratio that is related to a floating default barrier. This characteristic substantially deviates from the DOC option. A model risk is thus encountered. Second, there are many more parameters in the CDG model, and some of the parameters that are related to the process of the leverage ratio should be estimated using the book value of liabilities. In the best situation, we can only use quarterly data, and therefore the quality of estimation is of great concern in both the MLE and proxy approaches.

From the empirical result of the LS model, we learn that structural models may be improved by considering a floating default barrier. The CDG model is exactly designed for this purpose. However, it contains too many parameters to be estimated, and misses the modeling of equity. Thus, we cannot use high-frequency equity data to estimate the parameters of the process for the leverage ratio. These two undesirable features together make the CDG model less useful in predicting corporate bond prices and difficult to test statistically. In future, a parsimonious structural model should be constructed that incorporates a soft barrier and equity value modeling.

The CDG model, shown in Table 6 and 7, consistently underestimates the yields for all ratings and bond maturities. As the LS model may suffer from the extreme overestimation of short-term bond yields in theory, the CDG model is originally designed to pull down the short-term yields using a floating barrier. However, this ultimately pulls down corporate bond yields for all maturities. Our empirical study shows that the LS model already underestimates bond yields on average, even for short-term bonds, and thus pulling down the short-term bond yields may not be necessary, except when overestimation is extreme. The CDG model fails to improve the prediction of short-term bonds or maintain its predictive power for long-term bonds.

5 Conclusion

This paper investigates the systematic bias in the testing of structural models of corporate bond pricing using proxies and gives a bias-reduced empirical comparison of the Merton, LS, and CDG models. By option properties and simulation, we show that the sum of the market value of equities and the book value of liabilities is an upwardly biased estimator for the market value of a firm's assets. When this bias is carried forward to test structural models, there is a significant underestimation of corporate bond yields with structural models.

Apart from the proxy for firm value, we show that the dividend yields reported by CompuStat that is also biased upward leads to overestimation of asset payout ratio. If the reported dividend yield and the proxy firm value are put together in a barrier-dependent model, then the corporate bond yields for dividend-paying firms will be overestimated. We give empirical evidence for this claim.

Another important contribution of this paper is that it empirically examines the Merton, LS, and CDG models using maximum likelihood estimation. We find that the MLE approach improves the performance of the Merton and LS models, but not of the CDG model. We document that the LS model outperforms the Merton model in almost all aspects, especially for short-term, long-term, and low rating bonds. The CDG model performs the worst among the three models, as it suffers from a lack of relation between the market value of equity and the market value of the firm. Moreover, the CDG model involves too many parameters, and thus generates many difficulties in the estimation process.

Based on the empirical evidence, we give several suggestions for the development of structural models in the future. For the testing of structural models, we suggest that proxies should be chosen with a special care to avoid any systematic bias, and whenever possible, statistical estimation methods should be preferred. For the construction of structural models, we propose that a parsimonious model should be developed that incorporates a soft default barrier. Furthermore, a desirable model should clearly specify the relationship between the firm value and the equity value.

Appendix

A A proof of the bias of the proxy for firm value in the LS model

Let V be the true market value of assets, V_{proxy} be the proxy for firm value and S be the maket value of equity. The proxy for firm value then relates to the equity and liabilities by

$$V_{proxy} = S + X.$$

We view the market value of equity as a DOC option on the underlying asset V with a strike price X, default barrier H, and rebate R. Thus,

$$S = \text{DOC}(V, X, H, R).$$

The no arbitrage pricing principle shows that the DOC price must be greater than the intrinsic value if the barrier is set to the book value of liabilities, that is,

$$\text{DOC}(V, X, H, R) > V - X, \tag{A.1}$$

where $H = X$. If this is not the case (that is, if $V - \text{DOC}(V, X, X, R) - X \geq 0$), then an investor can make an arbitrage profit by selling the asset at V to purchase the DOC option. The remaining cash is put into a bank account. A profit can then be made by taking two different actions that correspond to two possible scenarios.

1. If the asset price V does not breach the barrier level X before maturity, then on the maturity day (T), the investor will exercise the option to purchase the asset for a value of X so that the investor's short position in the asset will be canceled. An arbitrage profit of

   $$[V - \text{DOC}(V, X, X)]\, e^{rT} - X$$

   is then made at time T.

2. If the asset value breaches the barrier level X at time $\tau < T$, then the investor will receive a rebate of R. The investor will purchase the asset from the market right away for an amount X to cancel the short position in the asset. An arbitrage profit of

   $$[V - \text{DOC}(V, X, X)]\, e^{r\tau} - X + R$$

   is then made at time τ.

This proves the inequality (A.1), which is a model-independent property of DOC options.

A consequence of the inequality is that

$$\text{DOC}(V_{proxy}, X, X, R) > V_{proxy} - X = S = \text{DOC}(V, X, X, R).$$

As the DOC option is an increasing function of the underlying asset price, the proxy firm value, V_{proxy}, is clearly larger than the true value, V, if the default barrier is set to X. This shows that the proxy firm value in the study of EHH is an upwardly biased estimator.

Actually, the inequality (A.1) holds for all $H < X$, because the DOC option is a decreasing function of the default barrier. Moreover, the difference $\text{DOC}(V, X, H, R) - (V - X)$ can be widened by decreasing the value of H, which implies that the smaller the default barrier the more significant the upward bias that is induced by the proxy for firm value. Therefore, it is the most significant bias in the Merton model.

B Likelihood function of the Merton model

The underlying asset price evolves as the Black-Scholes dynamics,

$$d \ln V_t = (\mu - \sigma^2/2)dt + \sigma dZ_t,$$

where V_t is the market value of assets at time t, μ is the drift of the business, σ is the asset volatility, and Z_t is a standard Wiener process. Under the physical probability measure, the density function of $\ln V_t$ is given by

$$g(v_i|v_{i-1}) = \frac{1}{\sigma\sqrt{2\pi(t_i - t_{i-1})}} \times \exp\left\{-\frac{[v_i - v_{i-1} - (\mu - \sigma^2/2)(t_i - t_{i-1})]^2}{2\sigma^2(t_i - t_{i-1})}\right\}.$$

The Merton model views the market value of equity S as a standard call option on the market value of assets V such that

$$S = V \cdot N(d_1) - Xe^{-rT} \cdot N(d_2),$$

where X is the book value of corporate liabilities, r is the risk-free rate, T is maturity, $N(\cdot)$ is the cumulative distribution function for a standard normal random variable and

$$d_1 = \frac{\ln(V/X) + (r + \sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = \frac{\ln(V/X) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

As inference is made based on the observable market values of equities, we formulate the log likelihood function of μ and σ by

$$L(\mu,\sigma)=\sum_{i=2}^{n}\ln f(S_i|S_{i-1},\mu,\sigma),\qquad S_i\equiv S_(t_i),$$

where $f(\cdot)$ denotes the probability density function of S and $S_(t_i)$ denotes the market value of equity at time t_i. After applying the standard change of variable technique, we obtain

$$f(S_i|S_{i-1},\mu,\sigma)=g(v_i|v_{i-1},\mu,\sigma)\times[V_i\cdot N(d_1)|_{V=V_i}]^{-1}.$$

Hence, the log-likelihood function reads

$$L(\mu,\sigma)=\sum_{i=2}^{n}\{\ln g(v_i|v_{i-1})-\ln[V_i\cdot N(d_1)|_{V=V_i}].$$

The MLE is the solution to the following optimisation problem.

$$\max_{\mu,\sigma} L(\mu,\sigma)\quad s.t.\quad S(t_i)=C(t_i,V(t_i),\sigma),\ \forall\, i=1,2,\cdots,n.$$

C DOC option pricing formula

$$\begin{aligned}DOC(V,X,H,R) &= VN(a)-Xe^{-rT}N\left(a-\sigma\sqrt{T}\right)\\ &- V(H/V)^{2\eta}N(b)+Xe^{-rT}(H/V)^{2\eta-2}N\left(b-\sigma\sqrt{T}\right)\\ &+ R(H/V)^{2\eta-1}N(c)+R(V/H)N\left(c-2\eta\sigma\sqrt{T}\right),\end{aligned}$$

where V is the market value of firm assets, X is the future promised payment, H is the barrier level, σ is the asset volatility, r is the risk-free interest rate, T is the time to maturity, R is the rebate paid to the equity holders upon default (asset value breaches the barrier), $N(\cdot)$ is the cumulative distribution function for a standard normal random variable, and

$$a=\begin{cases}\frac{\ln(V/X)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X\geq H,\\ \frac{\ln(V/H)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X<H,\end{cases}$$

$$
\begin{aligned}
b &= \begin{cases} \frac{\ln(H^2/VX)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X \geq H, \\ \frac{\ln(H/V)+(r+\sigma^2/2)T}{\sigma\sqrt{T}}, & \text{for } X < H, \end{cases} \\
c &= \frac{\ln(H/V)+(r+\sigma^2/2)T}{\sigma\sqrt{T}} \quad \text{and} \quad \eta = \frac{r}{\sigma^2} + \frac{1}{2}.
\end{aligned}
$$

D Pricing formulas of the structural models

D.1 The Merton model

The original Merton model considers a corporate zero-coupon bond with a maturity T and face value X. The model assumes a constant interest rate r and market values of assets V_t follow a geometric Brownian motion, i.e.

$$dV_t = (\mu - \delta)V_t dt + \sigma V_t dW_{1t}, \tag{D.2}$$

where μ, δ and σ is the drift, payout ratio and volatility of market values of assets respectively and W_{1t} is a standard Brownian motion.

Assuming no intermediate default, the terminal payoff of the bond is the minimum of the face amount of the bond and the market value of assets at maturity V_T. By discounting it under the risk neutral measure, the corporate bond price is expressed as a risk-free bond minus a put option on the underlying assets V with a strike price of X and maturity T, that is,

$$
\begin{aligned}
BP_c^M(V_0, X, T) &= Xe^{-rT} - P(V_0, X, T) \\
&= Xe^{-rT}N(d_2) + V_0 e^{-\delta T}N(-d_1),
\end{aligned} \tag{D.3}
$$

where

$$d_1 = \frac{\ln(V_0/X) + (r - \delta + \frac{\sigma^2}{2})T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{T}.$$

As the original Merton model only deals with a zero coupon bond, EHH propose the extended Merton model to treat a coupon bearing bond as a portfolio of zero coupon bonds. Default is assumed to occur only at coupon paying dates when the market value of assets is less than a default barrier K. Upon default, bondholders receive a portion of market values of assets, the recovery rate ω. The pricing formula of the extended Merton model is found to be

$$
\begin{aligned}
BP_c^{EM}(V_0, X, T) &= \sum_{i=1}^{n-1} e^{-rt_i} E^Q\left[(c/2)I_{\{V_{t_i} \geq K\}} + \min(wc/2, V_{t_i})I_{\{V_{t_i} < K\}}\right] \\
&+ e^{-rT} E^Q\left[(1 + c/2)I_{\{V_T \geq K\}} + \min(w(1 + c/2), V_T)I_{\{V_T < K\}}\right],
\end{aligned} \tag{D.4}
$$

where c is the coupon rate,

$$
\begin{aligned}
E^Q[I_{\{V_t \geq K\}}] &= N(d_2(K,t)), \\
E^Q[I_{\{V_t < K\}} \min(\psi, V_t)] &= V_0 e^{(r-\delta)t} N(-d_1(\psi,t)) + \psi\left[N(d_2(\psi,t)) - N(d_2(K,t))\right], \\
d_1(x,t) &= \frac{\ln(V_0/x) + (r - \delta + \sigma^2/2)t}{\sigma\sqrt{t}}, \\
d_2(x,t) &= d_1(x,t) - \sigma\sqrt{t}.
\end{aligned}
$$

In formula (D.4), we assume n coupon paying dates of $\{t_1, t_2, \cdots, t_n\}$, that $t_n = T$, and use $N(\cdot)$ to represent the cumulative distribution function of a standard normal random variable.

D.2 The LS model

For the LS model, asset prices are assumed to follow equation (D.2), and interest rates r_t are assumed to be stochastic with dynamics of

$$dr_t = (\alpha - \beta r_t)dt + \eta dW_{2t},$$

or, equivalently,

$$dr_t = \kappa(\theta - r_t)dt + \eta dW_{2t}, \tag{D.5}$$

where α, β, η, κ and θ are some parameters and W_2 is another standard Brownian motion process. The underlying asset price and the interest rate are correlated processes with correlation coefficient ρ.

Under the LS framework, default occurs if the market value of assets at time t (V_t) reaches a threshold value K, or equivalent $L_t = V_t/K$ reaches one. Hence, the pricing formula for a corporate zero coupon bond can be calculated as

$$BP_c^{LS}(L_0, r_0, T) = D'(r_0, T)[1 - \omega Q(L_0, r_0, T)], \tag{D.6}$$

where

$$
\begin{aligned}
Q(L_0, r_0, T, n) &= \sum_{i=1}^{n} q_i, \\
q_1 &= N(a_1), \\
q_i &= N(a_i) - \sum_{j=1}^{i-1} q_j N(b_{ij}), \qquad i = 2, 3, ..., n,
\end{aligned}
$$

$$
\begin{aligned}
a_i &= \frac{-\ln X - M(iT/n, T)}{\sqrt{S(iT/n)}}, \\
b_{ij} &= \frac{M(jT/n, T) - M(iT/n, T)}{\sqrt{S(iT/n) - S(jT/n)}},
\end{aligned}
$$

and

$$
\begin{aligned}
M(t, T) &= \left(\frac{\alpha - \rho\sigma\eta}{\beta} - \frac{\eta^2}{\beta^2} - \frac{\sigma^2}{2} - \delta\right) t \\
&+ \left(\frac{\rho\sigma\eta}{\beta^2} + \frac{\eta^2}{2\beta^3}\right) \exp(-\beta T)[\exp(\beta t) - 1] \\
&+ \left(\frac{r_0}{\beta} - \frac{\alpha}{\beta^2} + \frac{\eta^2}{\beta^3}\right) [1 - \exp(-\beta t)] \\
&- \left(\frac{\eta^2}{2\beta^3}\right) \exp(-\beta T)[1 - \exp(-\beta t)], \\
S(t) &= \left(\frac{2\rho\sigma\eta}{\beta} + \frac{\eta^2}{\beta^2} + \sigma^2\right) t \\
&- \left(\frac{2\rho\sigma\eta}{\beta^2} + \frac{2\eta^2}{\beta^3}\right) [1 - \exp(-\beta t)] \\
&+ \left(\frac{\eta^2}{2\beta^3}\right) [1 - \exp(-2\beta t)],
\end{aligned}
$$

where $D'(r_0, T)$ is the price of a zero coupon bond with a face value of \$1 and time to maturity T under interest rates that follow the Vasicek (1977) model, and $N(\cdot)$ is the cumulative density function of a standard normal distribution. When n tends to infinity, the term $Q(L_0, r_0, T)$ is the limit of $Q(L_0, r_0, T, n)$ and thus we can calculate the corporate bond price predicted by the LS model.

The pricing formula for a corporate coupon bearing bond is simply the sum of all of the individual zero coupon bonds, that is,

$$
BP_c^{LS}(L_0, r_0, T) = \sum_{i=1}^{n} D'(r_0, t_i) \cdot X_i \cdot [1 - \omega Q(L_0, r_0, t_i)], \tag{D.7}
$$

$$
\text{and} \quad X_1 = X_2 = \ldots = X_{n-1} = \frac{Xc}{2}, \quad X_n = X(1 + \frac{c}{2}).
$$

D.3 The CDG model

The CDG model assumes that the asset price and interest rate follow equations (D.2) and (D.5), respectively. Moreover, the log-default threshold ($k_t = \ln K_t$) evolves as

$$dk_t = \lambda[\ln V_t - \nu - \phi(r_t - \theta) - k_t]dt, \tag{D.8}$$

where λ, ν and ϕ are constant parameters. A default event occurs if the market value of assets hits the threshold value at time t or, equivalently, if $l_t = k_t - \ln(V_t)$ is equal to zero. The pricing formula for a corporate zero coupon bond is obtained as:

$$BP_c^{CDG}(l_0, r_0, T) = D'(r_0, T)[1 - \omega Q(l_0, r_0, T)], \tag{D.9}$$

where $D'(r_0, T)$ is the Vasicek (1977) price of a zero coupon bond with a face value \$1 and time to maturity T, and ω is the recovery rate.

By discretizing the time interval [0,T], the CDG shows that

$$\begin{aligned}
Q(l_0, r_0, T) &= \sum_{j=1}^{n_T} \sum_{i=1}^{n_r} q(r_i, t_j), \\
q(r_i, t_1) &= \Delta r \Psi(r_i, t_1) \qquad i = 1, 2, ..., n_r, \\
q(r_i, t_j) &= \Delta r \left[\Psi(r_i, t_j) - \sum_{v=1}^{j-1} \sum_{u=1}^{n_r} q(r_u, t_v) \psi(r_i, t_j | r_u, t_v) \right] \\
& \qquad i = 1, 2, ..., n_r \text{ and } j = 2, 3, ..., n_T, \\
\Psi(r, t) &= \pi(r_t, t | r_0, 0) N \left(\frac{\mu(r_t, t | l_0, r_0, 0)}{\Sigma(r_t, t | l_0, r_0, 0)} \right), \\
\psi(r_t, t | r_s, s) &= \pi(r_t, t | r_s, s) N \left(\frac{\mu(r_t, t | l_s = 0, r_s, s)}{\Sigma(r_t, t | l_s = 0, r_s, s)} \right), \\
\mu(r_t, t | l_s, r_s, s) &= E_s^T[l_t] + \frac{\text{cov}_s^T[l_t, r_t]}{\text{var}_s^T[r_t]} (r_t - E_s^T[r_t]), \\
\Sigma(r_t, t | l_s, r_s, s) &= \sqrt{\text{var}_s^T[l_t] - \frac{\text{cov}_s^T[l_t, r_t]^2}{\text{var}_s^T[r_t]}},
\end{aligned}$$

where

$$\begin{aligned}
l^Q(r) &= \frac{\delta + \frac{\sigma^2}{2}}{\lambda} - \nu + \phi\theta - r \left(\frac{1}{\lambda} + \phi \right), \\
B_\kappa^{(u)} &= \frac{1 - e^{-\kappa u}}{\kappa},
\end{aligned}$$

$$
\begin{aligned}
E_u^T[l_t] &= l_u e^{-\lambda(t-u)} - (1+\lambda\phi)\left(r_u + \frac{\eta^2}{\kappa^2} - \theta\right) e^{-\kappa(t-u)} B_{(\lambda-\kappa)}^{(t-u)} \\
&- \left(\frac{\eta\rho\sigma}{\kappa} + (1+\lambda\phi)\frac{\eta^2}{2\kappa^2}\right) e^{-\kappa(T-t)} B_{(\lambda+\kappa)}^{(t-u)} \\
&+ (1+\lambda\phi)\frac{\eta^2}{2\kappa^2} e^{-\kappa(T-t)} e^{-2\kappa(t-u)} B_{(\lambda-\kappa)}^{(t-u)} \\
&+ \left(\frac{\eta\rho\sigma}{\kappa} + \lambda\bar{l}^Q(0) - (1+\lambda\phi)\left(\theta - \frac{\eta^2}{\kappa^2}\right)\right) B_{\lambda}^{(t-u)}, \\
E_u^T[r_t] &= r_u e^{-\kappa(t-u)} + \left(\theta\kappa - \frac{\eta^2}{\kappa^2}\right) B_{\kappa}^{(t-u)} + \frac{\eta^2}{\kappa} e^{-\kappa(T-t)} B_{2\kappa}^{(t-u)}, \\
\mathrm{var}_u^T[l_t] &= \left(\frac{(1+\lambda\phi)\eta}{\lambda-\kappa}\right)^2 B_{2\kappa}^{(t-u)} \\
&+ \left[\sigma^2 + \left(\frac{(1+\lambda\phi)\eta}{\lambda-\kappa}\right)^2 - \left(2\frac{\rho\sigma(1+\lambda\phi)\eta}{\lambda-\kappa}\right)\right] B_{2\lambda}^{(t-u)} \\
&+ 2\left[\left(\frac{\rho\sigma(1+\lambda\phi)\eta}{\lambda-\kappa}\right) - \left(\frac{(1+\lambda\phi)\eta}{\lambda-\kappa}\right)^2\right] B_{(\lambda+\kappa)}^{(t-u)}, \\
\mathrm{var}_u^T[r_t] &= \eta^2 B_{2\kappa}^{(t-u)}, \\
\mathrm{cov}_u^T[l_t, r_t] &= -\frac{(1+\lambda\phi)\eta^2}{\lambda-\kappa} B_{2\kappa}^{(t-u)} - \left(\sigma\eta\rho - \left(\frac{(1+\lambda\phi)\eta^2}{\lambda-\kappa}\right)\right) B_{(\lambda+\kappa)}^{(t-u)}.
\end{aligned}
$$

Herein, $N(\cdot)$ is the cumulative distribution function of a standard normal random variable, and $\pi(r_t, t|r_s, s)$ is the well-known transition density for a one-factor Markovian Gaussian interest-rate process.

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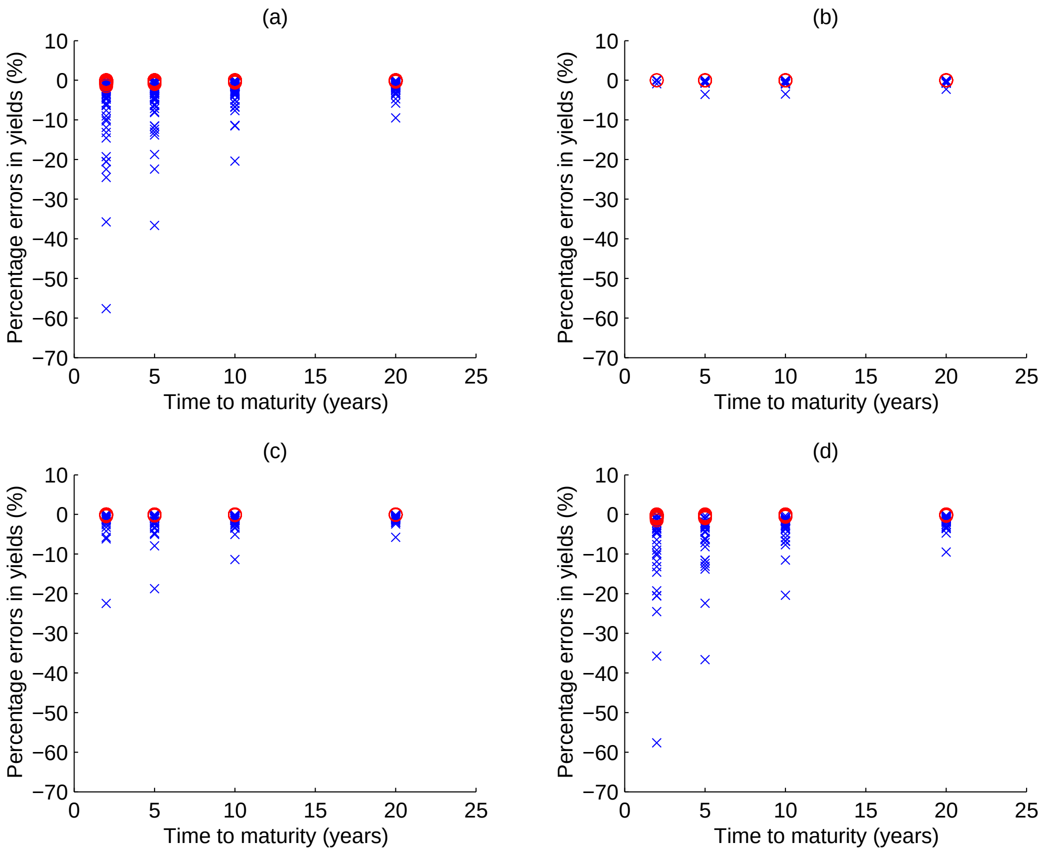

Figure 1: Simulation result of the Merton model. In all of the figures, 'o' indicates the percentage error in the yields using the MLE and 'x' indicates the percentage error using the proxy. Figure 2a plots the results for all bonds. Figures 2b, 2c, and 2d plot the results by high, medium, and low credit qualities, respectively.

Table 1: Simulation results for the Merton model

	MLE approach			Proxy approach		
	% error in prices	% error in yields	% error in spreads	% error in prices	% error in yields	% error in spreads
Characteristics	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)
Panel A: Different levels of coupon rate:						
c = 0%	0.06%	-0.10%	-4.14%	1.37%	-2.53%	-93.22%
	(0.10%)	(0.18%)	(3.08%)	(2.04%)	(5.48%)	(12.81%)
c = 8%	0.03%	-0.09%	-4.09%	0.75%	-2.35%	-92.28%
	(0.05%)	(0.18%)	(3.96%)	(1.26%)	(5.64%)	(14.64%)
Panel B: Different levels of total liabilities:						
X = 0.3	0.00%	-0.01%	-2.92%	0.15%	-0.22%	-86.89%
	(0.01%)	(0.01%)	(3.84%)	(0.31%)	(0.48%)	(20.98%)
X = 0.5	0.03%	-0.05%	-4.08%	0.77%	-1.64%	-94.68%
	(0.04%)	(0.07%)	(2.56%)	(1.05%)	(3.08%)	(7.12%)
X = 0.7	0.10%	-0.22%	-5.35%	2.25%	-5.45%	-96.69%
	(0.11%)	(0.26%)	(3.69%)	(2.33%)	(8.28%)	(4.88%)
Panel C: Different levels of time to maturity:						
T = 2	0.03%	-0.15%	-4.50%	0.70%	-4.21%	-93.77%
	(0.05%)	(0.28%)	(5.08%)	(1.44%)	(8.95%)	(22.86%)
T = 5	0.05%	-0.11%	-3.86%	1.18%	-3.06%	-95.34%
	(0.07%)	(0.18%)	(2.66%)	(1.86%)	(5.36%)	(5.70%)
T = 10	0.05%	-0.07%	-3.83%	1.26%	-1.71%	-91.83%
	(0.09%)	(0.11%)	(2.71%)	(1.84%)	(2.69%)	(8.66%)
T = 20	0.06%	-0.04%	-4.27%	1.10%	-0.77%	-90.08%
	(0.10%)	(0.06%)	(3.15%)	(1.69%)	(1.12%)	(10.67%)

A percentage error is calculated as the estimated value minus the true value divided by the true value.

Table 2: Simulation results for the LS model

	MLE approach			Proxy approach		
	% error in prices	% error in yields	% error in spreads	% error in prices	% error in yields	% error in spreads
Characteristics	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)
Panel A: Different levels of coupon rate:						
c = 0%	0.07%	-0.09%	-0.73%	5.46%	-5.95%	-51.57%
	(0.13%)	(0.21%)	(2.76%)	(7.54%)	(9.79%)	(39.53%)
c = 8%	0.06%	-0.10%	-0.63%	4.32%	-6.58%	-55.01%
	(0.11%)	(0.22%)	(0.93%)	(6.90%)	(10.46%)	(39.50%)
Panel B: Different levels of total liabilities:						
X = 0.3	0.01%	-0.01%	-0.88%	2.29%	-2.53%	-69.45%
	(0.02%)	(0.02%)	(3.29%)	(2.76%)	(2.65%)	(25.52%)
X = 0.5	0.04%	-0.06%	-0.50%	6.24%	-8.84%	-64.69%
	(0.05%)	(0.08%)	(0.90%)	(5.04%)	(7.57%)	(25.77%)
X = 0.7	0.13%	-0.21%	-0.67%	6.14%	-7.42%	-25.74%
	(0.18%)	(0.33%)	(1.03%)	(10.72%)	(14.91%)	(47.22%)
Panel C: Different levels of time to maturity:						
T = 2	0.04%	-0.15%	-1.31%	2.88%	-8.19%	-74.88%
	(0.10%)	(0.34%)	(3.91%)	(6.90%)	(15.49%)	(45.48%)
T = 5	0.06%	-0.11%	-0.69%	4.37%	-7.06%	-61.68%
	(0.13%)	(0.21%)	(0.83%)	(7.38%)	(9.99%)	(39.11%)
T = 10	0.07%	-0.07%	-0.44%	5.77%	-5.64%	-45.96%
	(0.13%)	(0.13%)	(0.56%)	(7.34%)	(6.51%)	(31.79%)
T = 20	0.07%	-0.05%	-0.28%	6.53%	-4.16%	-30.65%
	(0.12%)	(0.09%)	(0.40%)	(6.87%)	(4.56%)	(23.64%)

Table 3: Summary statistics of the corporate bond sample

Panel A

Characteristics	Mean	S.D.	Minimum	Median	Maximum
Time to maturity (T)	9.91	8.03	1.04	7.75	49.95
Coupon rate (c)	8.20	1.52	0	8.5	15
Yield-to-maturity (y)	7.68	1.54	3.94	7.48	22.49
Moody's ratings	7.24	2.73	2	7	24
S&P ratings	6.99	2.67	2	7	16
Market capitalisation (MV)	7450.66	10733.12	230.55	3428.44	95983.1
Total liabilities (X)	5151.77	10728.75	113.6	2324.49	150424.59

Panel B

Year	Number of bonds	T	c	y	Moody's ratings*	S&P ratings*	MV	X
1986	20	11.47	9.75	8.17	6.95	6.65	4479.68	4622.74
1987	29	10.46	9.18	9.55	5.93	5.93	6309.20	5575.82
1988	47	8.08	9.07	10.02	6.45	6.26	5286.23	9584.63
1989	52	8.48	9.11	8.93	6.69	6.46	6355.56	8661.61
1990	49	9.26	9.16	9.14	6.31	6.27	8371.26	10086.37
1991	68	10.89	8.91	7.47	6.46	6.25	8573.71	5124.63
1992	77	10.19	8.43	7.38	7.25	6.77	6892.26	4050.76
1993	94	10.21	7.67	6.41	7.30	6.90	7572.97	4120.06
1994	99	9.62	7.75	8.72	7.55	7.17	7752.24	4518.75
1995	138	10.02	7.63	6.40	7.66	7.43	8107.86	4203.99
1996	134	10.23	7.55	7.05	8.07	7.95	7754.13	3231.63

*For the Moody's rating, 1 stands for Aaa+, 2 stands for Aaa, and so on. For the S&P ratings, 1 stands for AAA+, 2 stands for AAA, and so on. For both rating systems, 24 stands for NR, which means that the bond is not rated.

Table 4: Overall empirical results for the Merton, LS, and CDG model

Method of estimation	Mean percentage error in price (Standard deviation)	Mean percentage error in yield (Standard deviation)	Mean yield difference (Standard deviation)
Panel A: Empirical results of different models using the MLE approach:			
Merton	2.37%	-1.82%	-0.03%
	(8.78%)	(35.34%)	(3.21%)
LS	3.57%	-4.38%	-0.28%
	(6.15%)	(14.43%)	(1.29%)
CDG	5.71%	-12.19%	-0.98%
	(7.05%)	(17.81%)	(1.68%)
Panel B: Empirical results of different models using the proxy approach:			
Merton	7.22%	-15.17%	-1.26%
	(6.12%)	(10.45%)	(1.07%)
LS	6.19%	-9.45%	-0.73%
	(5.53%)	(8.69%)	(0.92%)
CDG	5.40%	-10.83%	-0.87%
	(8.22%)	(21.53%)	(2.06%)

A percentage error in the price is calculated as the model price minus the market price divided by the market price. Similar calculations apply to other quantities. The yield difference is obtained by subtracting the market yield from the model yeild.

Table 5: Empirical results of the LS model when the default barrier is set to the book value of liabilities

	MLE approach			Proxy approach		
	% error in prices	% error in yields	Yield differences	% error in prices	% error in yields	Yield differences
Characteristics	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)
Panel A: Empirical results with different ratings:						
High ratings	-3.71%	13.76%	1.13%	1.68%	-0.41%	0.01%
	(11.41%)	(38.11%)	(2.89%)	(6.54%)	(17.27%)	(1.29%)
Medium ratings	-3.74%	15.75%	1.41%	2.39%	-1.91%	-0.05%
	(11.01%)	(37.38%)	(3.38%)	(7.00%)	(16.71%)	(1.49%)
Low ratings	-6.84%	42.66%	4.22%	8.49%	-10.38%	-1.02%
	(15.58%)	(79.12%)	(7.79%)	(12.73%)	(33.01%)	(3.64%)
Panel B: Empirical results with different maturities:						
Short maturity	-2.13%	19.33%	1.72%	2.36%	-3.90%	-0.23%
	(10.84%)	(61.39%)	(5.16%)	(4.61%)	(23.84%)	(1.94%)
Medium maturity	-3.90%	14.31%	1.18%	2.13%	-0.46%	-0.01%
	(11.03%)	(30.66%)	(2.49%)	(7.47%)	(16.07%)	(1.41%)
Long maturity	-6.42%	13.12%	1.16%	1.91%	1.01%	0.11%
	(13.41%)	(22.40%)	(1.95%)	(9.41%)	(13.00%)	(1.10%)

Table 6: Empirical results of the Merton, LS, and CDG models by rating

	MLE approach			Proxy approach:		
	% error in prices	% error in yields	Yield differences	% error in prices	% error in yields	Yield differences
Characteristics	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)
Panel A: Empirical results for the Merton model:						
High rating	2.69%	-3.83%	-0.27%	6.42%	-13.38%	-1.07%
	(8.12%)	(31.37%)	(2.45%)	(4.90%)	(9.56%)	(0.76%)
Medium rating	3.37%	-4.69%	-0.22%	7.86%	-17.11%	-1.36%
	(8.24%)	(27.94%)	(2.67%)	(4.93%)	(7.53%)	(0.69%)
Low rating	-6.43%	40.13%	4.32%	16.71%	-34.26%	-3.58%
	(14.38%)	(73.82%)	(8.39%)	(14.04%)	(12.95%)	(2.62%)
Panel B: Empirical results for the LS model:						
High rating	3.11%	-3.62%	-0.22%	5.34%	-7.61%	-0.56%
	(5.66%)	(12.87%)	(1.06%)	(4.05%)	(6.63%)	(0.47%)
Medium rating	4.24%	-6.04%	-0.36%	6.95%	-11.53%	-0.84%
	(5.76%)	(13.56%)	(1.20%)	(4.50%)	(7.54%)	(0.63%)
Low rating	7.90%	-9.23%	-0.86%	15.94%	-28.79%	-2.88%
	(11.17%)	(30.51%)	(3.24%)	(13.54%)	(13.82%)	(2.65%)
Panel C: Empirical results for the CDG model:						
High rating	5.19%	-11.33%	-0.90%	4.68%	-9.73%	-0.77%
	(5.57%)	(13.95%)	(1.11%)	(6.89%)	(16.11%)	(1.33%)
Medium rating	6.23%	-14.60%	-1.14%	5.98%	-12.87%	-0.98%
	(5.18%)	(11.18%)	(0.95%)	(7.62%)	(21.03%)	(1.85%)
Low rating	11.42%	-15.54%	-1.49%	13.89%	-19.30%	-2.03%
	(19.94%)	(54.58%)	(5.91%)	(18.36%)	(60.18%)	(6.69%)

Table 7: Empirical results of the Merton, LS, and CDG models by maturity

	MLE approach			Proxy approach		
	% error in prices	% error in yields	Yield differences	% error in prices	% error in yields	Yield differences
Characteristics	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)	Mean (S.D.)
Panel A: Empirical results for the Merton model:						
Short maturity	-1.24%	6.78%	0.80%	3.76%	-18.18%	-1.52%
	(9.84%)	(59.09%)	(5.40%)	(2.85%)	(13.86%)	(1.31%)
Medium maturity	2.63%	-4.83%	-0.32%	7.23%	-14.18%	-1.16%
	(7.00%)	(16.78%)	(1.44%)	(6.03%)	(8.36%)	(1.02%)
Long maturity	7.16%	-7.34%	-0.56%	12.41%	-13.07%	-1.09%
	(8.65%)	(10.94%)	(0.97%)	(6.29%)	(7.92%)	(0.66%)
Panel B: Empirical results for the LS model:						
Short maturity	2.70%	-5.88%	-0.35%	3.89%	-10.62%	-0.79%
	(4.05%)	(18.63%)	(1.64%)	(2.77%)	(9.63%)	(0.98%)
Medium maturity	4.00%	-4.66%	-0.32%	6.58%	-9.66%	-0.75%
	(5.97%)	(12.06%)	(1.11%)	(5.96%)	(8.54%)	(0.98%)
Long maturity	3.85%	-1.44%	-0.07%	8.67%	-7.17%	-0.58%
	(8.63%)	(12.02%)	(1.04%)	(6.22%)	(7.04%)	(0.58%)
Panel C: Empirical results for the CDG model:						
Short maturity	2.98%	-14.62%	-1.13%	2.60%	-11.79%	-0.92%
	(5.25%)	(24.77%)	(2.43%)	(6.11%)	(31.28%)	(3.07%)
Medium maturity	6.06%	-11.81%	-0.96%	5.83%	-11.23%	-0.91%
	(7.24%)	(15.13%)	(1.35%)	(8.10%)	(15.87%)	(1.49%)
Long maturity	8.93%	-9.49%	-0.79%	8.53%	-8.41%	-0.71%
	(7.45%)	(8.78%)	(0.77%)	(9.83%)	(14.49%)	(1.24%)

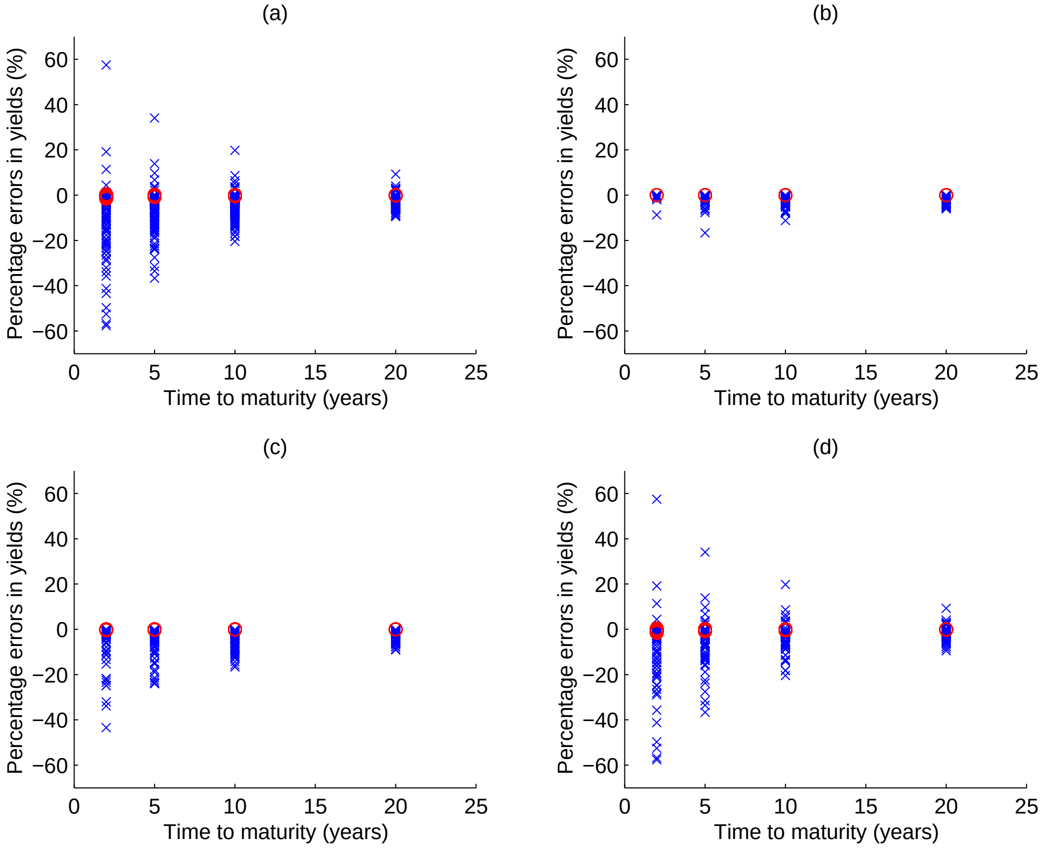

Figure 2: Simulation result of the LS model. In all of the figures, 'o' indicates the percentage error in the yields using the MLE and 'x' indicates the percentage error using the proxy. Figure 2a plots the results for all bonds. Figures 2b, 2c, and 2d plot the results by high, medium, and low credit qualities, respectively.

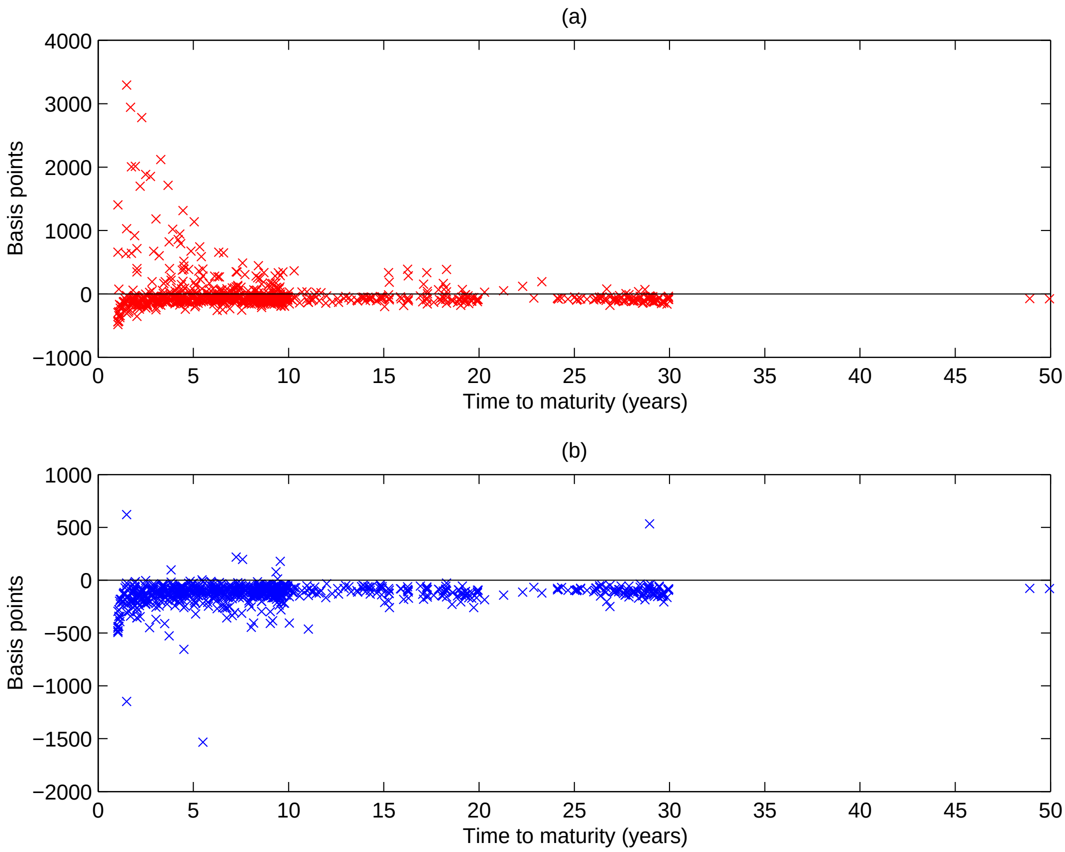

Figure 3: Empirical results of the Merton model. Figure 3a shows the errors in the yields using the MLE approach and Figure 3b shows the errors using the proxy approach.

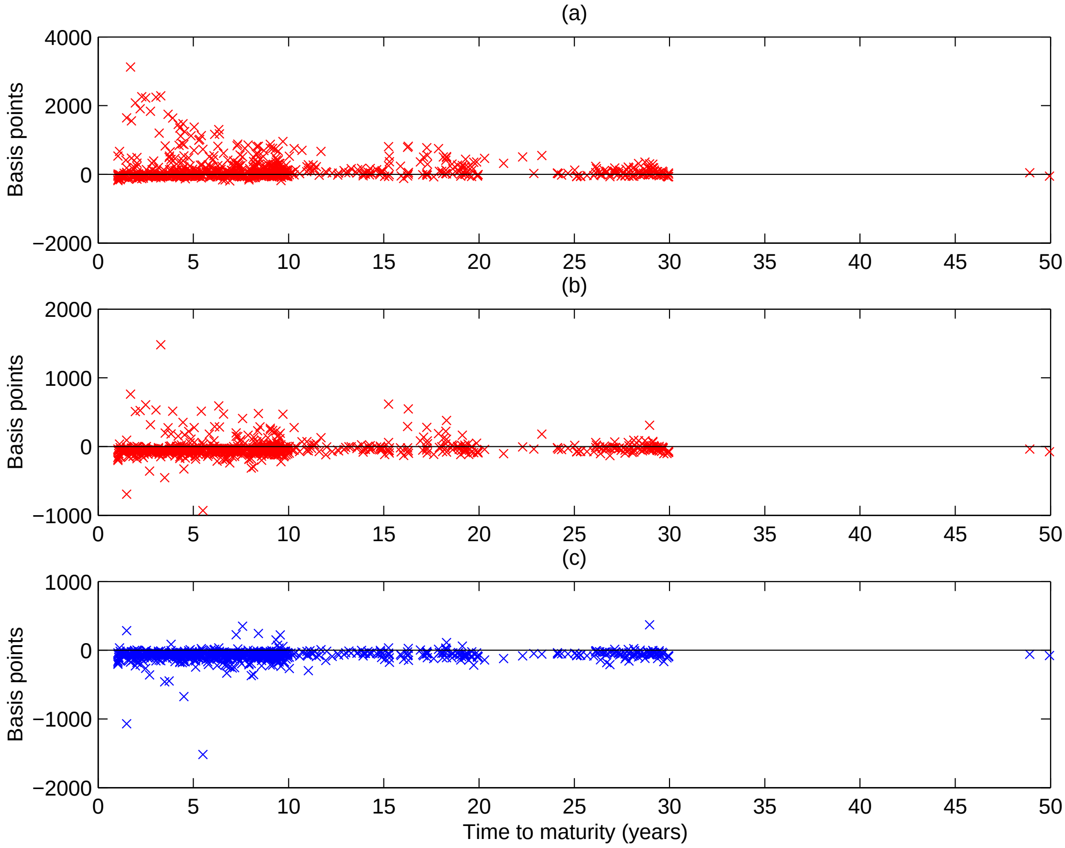

Figure 4: Empirical results of the LS model. Figure 4a shows the prediction errors in the yields using the MLE approach with the default barriers set to the total liabilities, and Figure 4b shows those with the default barriers set to the recovery value. Figure 4c shows the prediction errors using the proxy approach with the default barrier set to the recovery value.

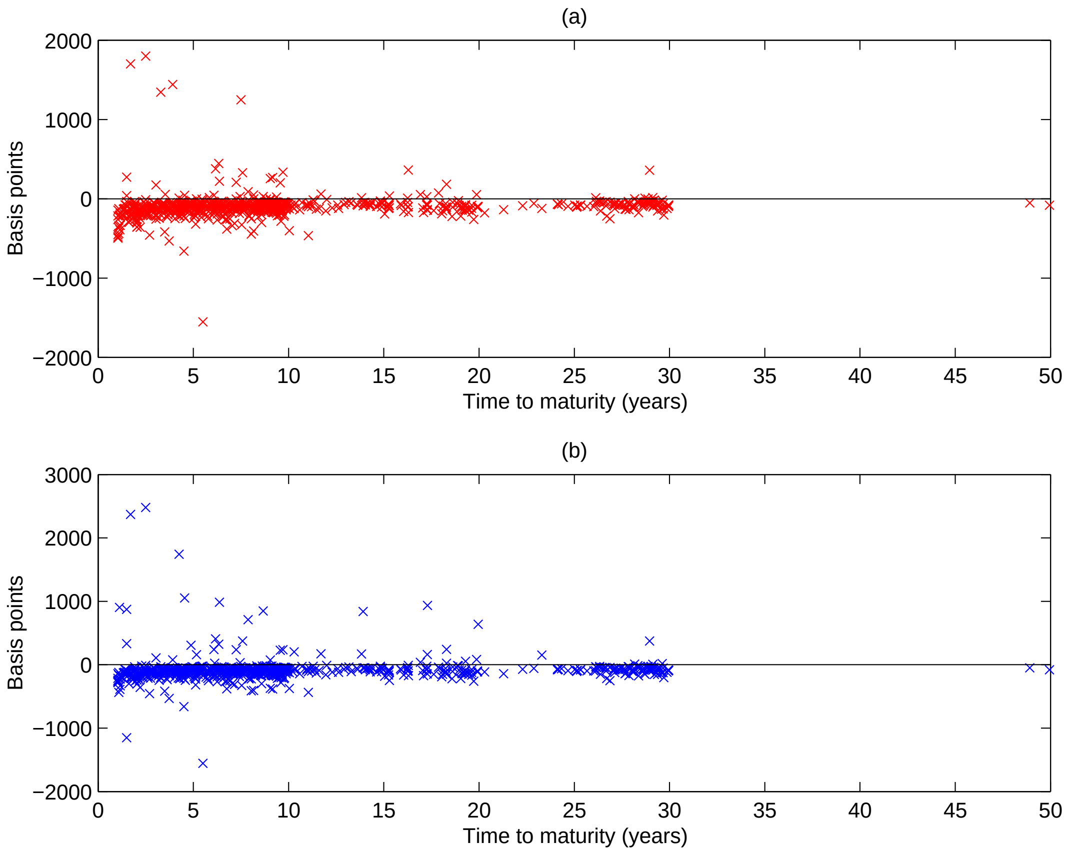

Figure 5: Empirical results of the CDG model. Figure 5a shows the errors in the yields using the MLE approach and Figure 5b shows the errors using the proxy approach.